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Reconocimiento del síndrome metabólico mediante una red neuronal

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RESUMEN

La enfermedad del Síndrome Metabólico(SM) según los estudios de la OMS últimamente crece y la *diabetes mellitus* en adultos aumentará en más del doble en 25 años y llegarán a 300 millones para el 2025.

Considerando que el noventa por ciento de pacientes que tienen *diabetes mellitus*, es muy probable que tengan síndrome metabólico, existen diferentes instituciones mundiales que investigan esta enfermedad, entre ellas: la OMS, Grupo de Estudio del Síndrome Metabólico de México, Asociación Americana del Corazón, Federación Internacional de Diabetes; cada una con un conocimiento establecido pero con ciertas diferencias, es decir, no hay un estándar de conocimiento para este síndrome. En vista que es un problema donde no existe conocimiento estándar, se propone realizar un sistema basado en redes neuronales para reconocer al síndrome metabólico, de tal manera que se considera cuatro organizaciones mundiales, se reconoce los criterios marcados, y se utilizará el algoritmo retro propagación(back propagation) de red neuronal para identificar al síndrome metábolico.

Se solicitó datos de historias clínicas de pacientes de los hospitales Dos de Mayo e Hipólito Unanue; luego se procedió a preparar la data en un archivo MS Excel para el entrenamiento del Perceptrón y se implementó el mismo usando Matlab; finalmente, la interfaz fue implementada en Java y conectada a Matlab para que pueda acceder al Perceptrón entrenado.

Palabras clave: síndrome metabólico; redes neuronales; algoritmo retro propagación; Matlab, Java.

ABSTRACT

The disease of metabolic syndrome (MS) according to WHO studies lately grows and diabetes mellitus in adults will more than double in 25 years and will reach 300 million by 2025.

Whereas ninety percent of patients with diabetes mellitus, is likely to have metabolic syndrome, different global institutions researching this disease, including: WHO Study Group Metabolic Syndrome Mexico, American Heart Association, International Diabetes Federation; each with established knowledge but with some differences, ie there is no standard of knowledge for this syndrome. Given that it is an issue where there is no standard knowledge, intends to make a system based on neural network to recognize the metabolic syndrome, so it is considered four world organizations, marked criteria recognized system, and retro algorithm used spread (back propagation) neural network to identify the metabolic syndrome.

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Data from medical records of patients from the Dos de Mayo and Hipolito Unanue Hospital was requested, then he proceeded to prepare the data in a file for MS Excel training Perceptron and it was implemented using Matlab. Finally, the interface was implemented in Java and Matlab connected so you can access the Perceptron trained.

Keywords: metabolic syndrome; neural networks; retro propagation algorithm; Matlab; Java.

INTRODUCCIÓN

El síndrome metabólico tiene sus antecedentes desde 1923, cuando Kylin describió la asociación de la hipertensión, hiperglucemia y gota como síndrome. Posteriormente varias otras anormalidades metabólicas se han asociado a este síndrome, incluyendo obesidad, microalbuminuria, y anormalidades en el fibrinolisis y la coagulación. Al síndrome también se le ha dado varios otros nombres: síndrome de resistencia a la insulina, síndrome plurimetabólico, cuarteto de la muerte [2].

En 1998, la Organización Mundial de la Salud (OMS), propuso una definición de la unificación para el síndrome y eligió llamarlo Síndrome Metabólico en lugar de Síndrome de resistencia a la insulina, nombre elegido porque no era considerado establecido que la resistencia de la insulina era la causa de todos los componentes del síndrome.

Además de la OMS, existen otras instituciones médicas mundiales que tratan acerca del Síndrome metabólico, como: Grupo de Estudio del Síndrome Metabólico de México (GESM) [2], Asociación Americana del Corazón (AAC) [3], Federación Internacional de Diabetes (IDF) [4], cada una con conocimientos dados, pero con ciertas o pocas diferencias, es decir no hay un estándar.

Redes neuronales artificiales (RNA) es un modelo matemático utilizado para la clasificación y diagnóstico en diversas áreas, eficaz en la toma de decisiones en el campo médico, y otros.

Xu *et al*, en la revista *Journal Biomedical Research*, en el trabajo titulado: "Aplicación de la propagación hacia atrás de errores de red neuronal artificial (BPANN) sobre las variantes genéticas en el PPAR-γ y el gen RXR-α y el riesgo de SM en una población china Han", han explorado las asociaciones entre los efectos de varios polimorfismos y el gen RXR- α y los factores ambientales , con el riesgo de síndrome metabólico por retro propagación de errores de red neuronal artificial (BPANN) entrenando la red para reconocer y clasificar patrones complejos en el SM [5].

Sumathy *et al*, en el 2010 en International Journal of Computer Applications trataron sobre diagnóstico de diabetes mellitus en base a los factores de riesgo. Para ello utilizaron una red multicapa supervisada con el algoritmo de aprendizaje de retropropagación haciendo una autopregunta a conciencia: "¿estoy teniendo diabetes mellitus o hipertensión?", de tal manera que el paciente va y comprueba su azúcar en la sangre [6].

Maitha H. Al Shamisi *et al*, leen datos de entrenamiento en un Excel desde Matlab, crean y entrenan un perceptrón multicapa al establecer un modelo de red neuronal

artificial para predecir la radiación solar usando Matlab [7].
El presente trabajo propuesto utiliza arquitectura de redes neuronales artificiales, una red multicapa que utiliza el algoritmo de aprendizaje de retropropagación, una interfaz implementada en Java y conectada a Matlab para que pueda acceder al perceptrón entrenado, y finalmente asignar diagnóstico positivo o negativo al reconocer el SM.

Breve estudio de la literatura

Muchos investigadores han hecho uso de las redes neuronales artificiales en diagnósticos de diversas áreas. Presentamos tres de ellos con una breve discusión.
Xu Zhao, Kang Xu [5] propusieron la aplicación del algoritmo de retropropagación en una RNA sobre las variantes genéticas en el PPAR-γ y el gen RXR-α y el riesgo de SM en una población china Han. Ellos indican que el síndrome metabólico, como prácticamente todas las enfermedades humanas, es el resultado de las interacciones entre los factores genéticos y ambientales.
Los factores ambientales como la obesidad, los bajos niveles de actividad física y los hábitos alimenticios inadecuados son fuertes determinantes del síndrome metabólico. Métodos estadísticos tradicionales debido a su propia limitación no muestran la verdadera relación entre el gen y elementos ambientales con el riesgo de síndrome metabólico, de donde se puede afirmar que el riesgo genético es bajo al reconocer la presencia del SM, y son más influyentes para la presencia del SM los factores ambientales.
Sumathy, Mythili Thirugnaman, Kumar, Jishnujit y Ranjith Kumar [6] propusieron en la arquitectura de la RNA 16 entradas, edad, género, los antecedentes familiares, la toma de medicamentos para la presión arterial alta (allí encontraron que tenían altos niveles de glucemia en un examen de salud en caso de enfermedad), fumar o usar productos de tabaco, la cantidad de consumo de verduras y frutas, la actividad física (30 minutos al día), el índice de masa corporal, relación cintura, cadera, aumento de la micción, el hambre, la sed, la mala cicatrización de las heridas, el estilo de vida (clase de trabajo, el trabajo sedentario, jubilados), diabetes gestacional, la ingesta frecuente de alimentos no vegetarianos, y picor por todo el cuerpo. 17 neuronas en la capa oculta. La neurona de salida con un valor de 0 o 1. El valor 0 representa al usuario que no se ve afectado con la diabetes y el valor 1 representa el usuario que sufre de la diabetes.
Maitha H. Al Shamisi, Ali Assi, y Hassan Hejase [7] presentaron en 5 pasos, colección de datos, preprocesamiento de la data, construcción de la red, entrenamiento y prueba al establecer un Modelo de red neuronal artificial para predecir la radiacion solar usando Matlab. Leen los datos de entrada desde un archivo Excel, con el comando Matlab xlsread ('archivo.xls','hoja_excel'), crean perceptrón multicapa con la orden newff (input,output,[i,…],{tf}), entrenan con el comando rain (MLP,input,output), la salida es predecir el promedio de radiación solar global mensual en la ciudad de Al Ain Emiratos Árabes Unidos.

De lo expuesto podemos concluir que las RNA, y el algoritmo de retropropagación son destacados en diagnósticos y predicciones.

PROCEDIMIENTO PARA DIAGNÓSTICO DE SÍNDROME METABÓLICO

Recolección de datos y valores de entrada

Un total de 140 historias clínicas de pacientes de los hospitales Dos de Mayo, Hipólito Unanue; otros centros, se utilizaron como entrenamiento (110) y validación (30) de la red neuronal. Las entradas para el sistema son paciente con antecedentes familiares de diabetes, sexo, edad > 45 años, presión arterial elevada ≥ 130 / 85 mmHg, colesterol total ≥ 200 mg/dL, triglicéridos en ayuno > 150 mg /dL, glucemia capilar en ayunas>100 mg/dL(5.6 mmol/L), intolerancia a la glucosa oral 140-199 mg/dL a las 2 hrs , HOMA IR=Insulina en ayunas (en mu por ml) x (glucemia en ayunas (en mg/dL)/18)/22.5, hiperuricemia, hiperinsulinemia, poliquistosis ovárica, HDL ♂ < 40 mg /dL, HDL ♀ < 50 mg / dL, LDL colesterol ≤ 165 mg/dL, VLDL colesterol, ICC ♂ ≥ 90 cms, ICC ♀ ≥ 80 cms, IMC ♂ ≥ 30 Kg /m^2 , IMC ♀ ≥ 27 Kg/m^2 , *acantosis nigricans*, antecedentes diabetes gestacional, productos macrosómicos, multiparidad o menopausia precoz, microalbuminuria > 20 μg/minuto, diabetes mellitus ADA ≥ 126 mg/d L (Glucemia ayunas), diabetes mellitus OMS ≥ 200 mg / d L, estado de prothrombotic, estado de proinflammatory, tabaquismo, sedentarismo menos de 30' minutos de actividad física x 5 días x semana, datos en Tabla I.

Los valores de entrada se tomaron en cuenta treinta y dos, considerando lo dado, por la OMS [1], Grupo de Estudio de Síndrome Métabólico de México (GESM), Asociación Americana del Corazón (AAC), e IDF (Federación Internacional de Diabetes), de donde al codificar en un cuadro se colocó "0" (Cuando no se verifica el hallazgo) o "1" (Si se verifica).

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Tabla I. Hallazgos para el síndrome metabólico

Hallazgos		OMS	GESM	AAC	IDF
Cod	Descripción				
.h_1	Paciente con antecedentes familiares de diabetes				
.h_2	Varón				
.h_3	Edad > 45 años				
.h_4	Presión arterial elevada ≥ 130 / 85 mmHg				
.h_5	Colesterol Total ≥ 200 mg/dL				
.h_6	Triglicéridos en ayuno > 150 mg /d L				
.h_7	Glucemia capilar en ayunas>100 mg/dL(5.6				
.h_8	Intolerancia a la glucosa oral 140-199 mg / d L a las 2 hrs				
.h_9	Intolerancia a la glucosa				
.h_{10}	HOMA IR=Insulina en ayunas (en mu por ml) x (glucemia en ayunas (en mg/dL)/18) /				
.h_{11}	Hiperuricemia				
.h_{12}	Hiperinsulinemia				
.h_{13}	Poliquistosis ovárica				
.h_{14}	HDL ♂ < 40 mg /d L				
.h_{15}	HDL ♀ < 50 mg / d L				
.h_{16}	LDL Colesterol ≤ 165 mg/dL				
.h_{17}	VLDL Colesterol				
.h_{18}	ICC ♂ ≥ 90 cms.				
.h_{19}	ICC ♀ ≥ 80 cms.				
.h_{20}	IMC ♂ ≥ 30 Kg /m ^{2}				
.h_{21}	IMC ♀ ≥ 27 Kg /m ^{2}				
.h_{22}	Acantosis nigricans				
.h_{23}	Antecedentes diabetes gestacional.				
.h_{24}	Productos macrosómicos.				
.h_{25}	Multiparidad o menopausia precoz				
.h_{26}	Microalbuminuria > 20 μg / minuto				
.h_{27}	Diabetes mellitus ADA ≥ 126 mg / d L (Glucemia ayunas)				
.h_{28}	Diabetes mellitus OMS ≥ 200 mg / d L				
.h_{29}	Estado de prothrombotic				
.h_{30}	*Estado de proinflammatory*				
.h_{31}	Tabaquismo				
.h_{32}	Sedentarismo. Menos de 30' minutos de actividad física x 5 dias x semana.				

Arquitectura de la red y entrenamiento

La arquitectura de la red se refiere al número de capas, número de nodos en cada capa, y el número de capas ocultas en la red. En el sistema propuesto, se escogieron 32 entradas para el reconocimiento del SM, de tal manera que habrá 32 capas de entrada para la red.
El doble, más o menos una cantidad escogida al azar es designado para la capa oculta, para el presente trabajo 10. La capa de salida de la red es una neurona con el valor cero o uno. El valor cero representa que el paciente no está afectado con el SM, y el valor 1 representa que se le reconoce en el paciente el SM.

Algoritmo retro propagación

En el presente trabajo propuesto damos los siguientes pasos, figura 1:

a. Leer de un primer archivo Excel la data de entrada
b. Leer del primer archivo Excel la data de salida que es de tipo entrenamiento
c. Creación del perceptrón multicapa
d. Error o tolerancia
e. Entrenamiento del perceptrón
f. Entrada de un archivo Excel para validar
g. Diagnósticos de un archivo Excel maestro para validar
h. Diagnósticos de un archivo Excel otro para validar
i. Consultar el perceptrón
j. Redondear
k. Comparativo del master
l. Comparativo de otro

```
clear;
clc;
a. bc_in=xlsread('hc_bc_110.xls','input');
b. bc_out=xlsread('hc_bc_110.xls','output');
c.pm=newff(bc_in,bc_out,[10],{'tansig','purelin'},'trainlm','lear
ngdm');
d.pm.trainParam.goal=0.01;
e. [pm,r]=train(pm,bc_in,bc_out);
f. test_in=xlsread('hc_test_30.xls','input');
g.diag_master=xlsread('hc_test_30.xls','output_master');
h.diag_otro=xlsread('hc_test_30.xls','output_otro');
i. res_pm=sim(pm,test_in);
j. res_pm_round=round(res_pm);
k. cmp_master=[res_pm_round;diag_master];
l. cmp_otro=[res_pm_round;diag_otro];
```

Figura 1. Comandos Matlab al ejecutar la RNA

Al comparar los resultados observamos un acierto del 55% en la figura 2.

Validación

Se validó el sistema usando 30 plantillas de historias clínicas de pacientes para su evaluación (cada paciente se acompañó por el diagnóstico del sistema experto, se presentaron a dos médicos especialistas para que hicieran su propio diagnóstico), los datos se muestran en la tabla 2.

```
>> cmp_master
cmp_master =  Columns 1 through 13
   1   1   0   1   1   1   0   1   1   0   1   1   0
   0   1   1   1   0   1   1   1   1   1   1   0   1
 Columns 14 through 26
   1   1   1   0   1   1  NaN   0   0   1   1   1   1
   1   1   0   0   1   1   0    1   0   0   1   1   1
 Columns 27 through 30
   1   1   1   1
   1   1   1   0
Observación 1: Primer dígito binario para el sistema, y el
segundo para el médico.
Observación 2: 18 aciertos de 30, sistema experto versus
medico master (60%)
>> cmp_otro
cmp_otro =  Columns 1 through 13
   1   1   0   1   1   1   0   1   1   0   1   1   0
   0   1   1   1   0   1   0   0   1   1   1   0   1
 Columns 14 through 26
   1   1   1   0   1   1  NaN   0   0   1   1   1   1
   0   0   0   0   0   1   0    1   0   0   1   1   1
 Columns 27 through 30
   1   1   1   1
   1   1   1   0
Observación 2: 15 aciertos de 30, sistema experto versus
medico otro (50%)

** En promedio sistema experto versus dos médicos 55%
**
```

Figura 2. Sistema experto usando red neuronal versus dos médicos

La interfaz para la captura de datos en Java e interconectar con Matlab, se muestra en la figura 3.

RESULTADOS

Los resultados del sistema experto usando redes neuronales se compararon con el diagnóstico de dos (2) médicos vía tres métricas utilizadas para validar sistemas inteligentes: exactitud, sensibilidad y especificidad (Acc, Sen y Spec), respectivamente, definidos de la siguiente manera:

$$\text{Acc} = (a + d)/(a + b + c + d)$$
$$\text{Sen} = a/(a + c)$$
$$\text{Spec} = d/(b + d)$$

Donde “a” es el número de casos positivos clasificados correctamente, “b” es el número de casos positivos que fueron mal clasificados, “c” es el número de casos negativos que son mal clasificados, y “d” es el número de casos negativos clasificados correctamente.

Tabla 2. Pruebas de validación

N°	Historia	Sistema	Médico	Médico
1	129684	+	-	-
2	123901	+	+	+
3	151767	-	+	+
4	205688	+	+	+
5	209289	+	-	-
6	212889	+	+	+
7	213089	-	+	-
8	800121	+	+	-
9	803053	+	+	+
10	805132	-	+	+
11	807981	+	+	+
12	807386	+	-	-
13	812284	-	+	+
14	825633	+	+	-
15	829556	+	+	-
16	829985	+	-	-
17	844913	-	-	-
18	845500	+	+	-
19	845785	+	+	+
20	848381		-	-
21	852383	-	+	+
22	855286	-	-	-
23	1206508	+	-	-
24	1206586	+	+	+
25	1209064	+	+	+
26	1209088	+	+	+
27	1209233	+	+	+
28	1210008	+	+	+
29	1210019	+	+	+
30	1210358	+	-	-

Entendemos por "positivo", que un caso, pertenece a un grupo de un correspondiente diagnóstico presuntivo, y por "negativo" que no pertenece.
La comparación resultante es representada en la tabla 3.
En la tabla 3 se muestra un acercamiento del rendimiento del sistema experto usando redes neuronales respecto a los resultados de los médicos, siendo relativamente bueno en las métricas de sensibilidad y exactitud, pero muy bajo en la especificidad, lo cual indica que para los casos positivos correctamente clasificados, es decir, para los que fueron diagnosticados con el síndrome metabólico, el sistema alcanzó un 76% y los médicos un 100%; mientras que para los casos negativos correctamente clasificados, es decir, para los que no fueron diagnosticados con el síndrome metabólico, los médicos con un 64%, acertaron mucho mejor que el sistema.

Figura 3. Interfaz de captura

Tabla 3. Resultados de la validación

Métricas	Médicos	Sistema Experto
Exactitud	83%	57%
Sensibilidad	100%	76%
Especificidad	64%	24%

CONCLUSIONES

Al término del presente trabajo podemos concluir que como no existe un conocimiento unificado para el diagnóstico del síndrome metabólico dado por las diversas organizaciones internacionales, se presenta una alternativa no convencional para reconocer el síndrome metabólico usando redes neuronales.
La alternativa está basada en la técnica del algoritmo de propagación en retroceso, teniendo como entradas los criterios hallazgos, pasando por el perceptrón multicapa, y obteniendo una neurona que identifica o no al síndrome metabólico.
El entrenamiento se realiza con historias clínicas en una muestra de ciento diez, y para la validación con una muestra de treinta, obteniendo un porcentaje satisfactorio del 55% en el entrenamiento, y de 57%, 76%, y 24% en la validación, siendo posible aumentar el resultado de la neurona de salida con los entrenamientos, para reconocer el síndrome metabólico.
Finalmente, podemos afirmar que el presente proyecto puede ser aplicado reuniendo a las otras organizaciones mundiales que también reconocen el síndrome metabólico, otras enfermedades, y se puede realizar otros trabajos como minimización de hallazgos criterios en el diagnóstico presuntivo de una enfermedad.

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Un modelo de optimización difuso para el problema de atraque de barcos

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RESUMEN

El problema de asignación de atraques (BAP) en un terminal marítimo de contenedores se define como la asignación factible de atraques a los barcos entrantes. En este trabajo, desarrollamos un modelo de optimización difusa para el BAP continuo y dinámico. Se asume que el tiempo de llegada de los barcos es impreciso, en el sentido de que los barcos pueden adelantarse o retrasarse hasta una tolerancia permitida. Se utiliza conjuntos difusos para representar la imprecisión en la llegada de los barcos. Para la solución del modelo se aplica el método de α – cortes. El modelo propuesto ha sido codificado en CPLEX y evaluado en diferentes instancias. Los resultados obtenidos muestran que el modelo propuesto puede ayudar a los administradores de un terminal marítimo de contenedores, pues tiene a su disposición planes de atraque con diferente grado de adelanto o retraso permitido y optimizados respecto al tiempo de espera.

Palabras clave: alpha – cortes; conjuntos difusos; imprecisión; optimización difusa; problema de asignación de atraques.

ABSTRACT

The berth allocation problem (BAP) in a maritime container terminal is defined as a feasible allocation of berths to incoming vessels. In this paper, we developed a fuzzy mathematical programming model for continuous and dynamic BAP. It is assumed that the arriving time of vessels is imprecise, in the sense that the vessels can have an advance or delay but only up to a permitted tolerance. Fuzzy sets are used to represent the imprecision. α – cuts method is applied to the model solution. The proposed model has been codified in CPLEX solver and evaluated in different instances. The obtained results shows that the proposed model can help the container terminal managers, since it has available berth plans with different degrees of allowed advance or delay, which are optimized according to the waiting time.

Keywords: alpha – cuts; fuzzy sets; imprecision; fuzzy optimizaticon; berth allocation problem.

INTRODUCCIÓN

Aproximadamente el 80% del mercado global se lleva a cabo a través del mar [1], y es mayormente transferido en contenedores. Los contenedores son cajas grandes de

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metal hechas en medidas estándar y se miden en múltiplos de 20 pies llamado "twenty-foot equivalent units" (TEU). El volumen del comercio en contenedores en el 2012 llegó a 155 millones de TEUs [2].

Los terminales portuarios que manejan contenedores son llamados terminales marítimos de contenedores (TMC), estos tienen diferentes y más complejas operaciones que los puertos de pasajeros o los puertos de carga y descarga a granel. Un TMC, generalmente sirve como zona de transbordo entre barcos y vehículos terrestres (trenes o camiones).

Los TMC, son sistemas abiertos con tres áreas diferenciadas (figura 1): el área de atraque (The berth area), donde los barcos son atracados para el servicio (cargar o descargar contenedores); el patio de almacenamiento de contenedores (The storage yard), donde los contenedores se almacenan temporalmente mientras esperan para ser exportados o importados; y el área de recepción y entrega (The gate area), que conecta el terminal de contenedores con el sistema de transporte al interior de un país (abastecimiento para la exportación o distribución en caso de importación). Cada uno de ellos presenta diferentes problemas de planificación y *scheduling* para ser optimizados [3], por ejemplo, la asignación de muelle, planificación de la estiba, *scheduling* de las grúas del muelle deben ser gestionados en el área de atraque; el problema de apilamiento de contenedores, las operaciones de transporte horizontal deben llevarse a cabo en el patio de contenedores.

En este trabajo se abordará el problema de asignación de atraques, también conocido como BAP (Berth Allocation Problem), un problema NP-duro de optimización combinatoria [4], que consiste en asignar a cada barco entrante una posición de atraque en el muelle.

Figura 1. Terminal marítimo de contenedores en el puerto de Valencia

Una vez que el barco llega al puerto, entra en tiempo de espera para atracar en el muelle. Los administradores de los TMC se enfrentan a dos decisiones relacionadas: *donde y cuando* los barcos deben atracar.
Los tiempos reales de la llegada de los barcos son bastante inciertos. Esta incertidumbre depende por ejemplo, de las condiciones meteorológicas (vientos, tormentas), problemas técnicos, otros terminales que el barco tiene que visitar o por otras razones. Los barcos pueden llegar respectivamente antes o después de su tiempo de llegada prevista ([5], [6]), solo la mitad de los barcos llega a tiempo [7]. Esto tiene efectos en las operaciones de carga y descarga, en otras actividades del terminal, y por tanto, en los servicios requeridos por el cliente.
Los administradores de los TMC cambian o revisan los planes, pero una revisión frecuente del plan de atraque no es deseable desde el punto de vista de la planificación de recursos [8]. Por lo tanto, la capacidad de adaptación del plan de atraque es importante para la buena performance del sistema que maneja un TMC. Como resultado, el posible adelanto o retraso en la llegada de los barcos, debe considerarse al hacer un plan de atraque.
Hay varios tipos de incertidumbre, como la aleatoriedad, imprecisión (ambigüedad, vaguedad), la confusión. Varios tipos de incertidumbre puede ser categorizados como estocásticos o difusos [9].
Los conjuntos difusos están especialmente concebidos para hacer frente a la imprecisión.
En este trabajo presentamos un modelo difuso para el BAP continuo y dinámico. Asumimos que el tiempo de llegada de los barcos es impreciso, en el sentido de que los barcos pueden adelantarse o retrasarse en su llegada. Esto puede ayudar a los tomadores de decisiones, pues el modelo permite obtener planes de atraque con diferente grado de imprecisión en las llegadas y optimizado respecto al tiempo de espera.

REVISIÓN DE LITERATURA

Hay varios atributos para clasificar los modelos relacionados con el BAP [10], los más importantes son: el espacial y el temporal.
El atributo espacial puede ser discreto o continuo. Para el caso discreto, el muelle es visto como un conjunto finito de atraques, donde cada atraque se describe por segmentos de longitud fija, usualmente, un atraque solo sirve a un barco a la vez; para el caso continuo, los barcos pueden atracar en posiciones arbitrarias dentro de los límites del muelle. El atributo temporal puede ser estático o dinámico. Para el caso estático, se asume que todos los barcos están en el puerto antes de realizar el plan de atraque; en el caso dinámico, los barcos pueden llegar al puerto en diferentes tiempos, durante el horizonte de planificación. Los barcos no pueden atracar antes de su hora prevista de llegada.
En ([10], [11]), los autores hacen una exhaustiva revisión de la literatura existente sobre el BAP. Hasta donde conocemos, en la literatura hay muy pocos estudios que traten el BAP con datos imprecisos.

Un trabajo que trató con entornos deterministas y difusos para la distribución de contenedores fue presentado en [12]. Los autores desarrollaron un modelo de programación binaria con parámetros difusos. Las distancias entre el muelle y la zona de terminales, el número de contenedores en un barco que ha llegado y la estimación del área disponible en cada terminal en un puerto, fueron asumidos en una condición imprecisa. La imprecisión se representó con números difusos trapezoidales. El objetivo es minimizar la distancia total recorrida por los contenedores desde el barco, hasta la zona de terminales que les fue asignado. El problema de asignación de atraques es tratado como un BAP discreto y estático y no consideran imprecisión en la llegada de los barcos.

Un modelo planteado como problema lineal entero mixto (MILP) difuso para el BAP discreto y dinámico fue propuesto en [13]. Los tiempos de llegada de los barcos están representados por números difusos triangulares. Se presenta el modelo y el diseño de un método de solución basada en MILP paramétrica, aunque no se muestra su evaluación. Pero, no tratan el BAP continuo, según Bierwirth [14], para el diseño de un modelo continuo, la planificación del atraque es más complicada que para un diseño discreto, pero se tiene la ventaja de una mejor utilización del espacio del muelle.

Un modelo de optimización difusa para el BAP continuo y dinámico fue propuesto en [15]. Los tiempos de llegada de los barcos están representados por números difusos triangulares, el modelo solo contempla posibles retrasos, pero no la posibilidad de adelantarse.

En este trabajo proponemos un modelo de optimización difusa para el BAP continuo y dinámico. Suponemos que los barcos pueden retrasarse o adelantarse un cierto tiempo tolerable o permitido. Esta tolerancia se representa con un conjunto difuso con función de pertenencia de tipo triangular.

El artículo se estructura de la siguiente forma. En la Sección 2, se presenta una revisión de la literatura relacionada con el BAP bajo imprecisión. Posteriormente, en la Sección 3, se describe los conceptos básicos para el desarrollo del trabajo. En la Sección 4, se propone el modelo de programación lineal difuso para el problema de asignación de atraques con imprecisión en la llegada de los barcos. Seguidamente, en la Sección 5, se evalúa los resultados obtenidos con el modelo. Por último, en la sección 6, se presenta las conclusiones y las líneas futuras de investigación.

PRELIMINARES

Los conjuntos difusos ofrecen un entorno flexible para optimizar sistemas complejos. A continuación se presenta algunos conceptos necesarios en el planteamiento de este trabajo.

Definición 1. Sea X el universo del discurso, un conjunto difuso $\tilde{A}$ en X es un conjunto de pares:

$$\tilde{\boldsymbol{A}} = \{(x, \mu_{\tilde{A}}(x)), x \in X\}$$

Donde $\mu_{\tilde{A}}: X \rightarrow [0,1]$ es llamada *función de pertenencia.*

$\mu_{\tilde{A}}(x)$ Representa el grado en que *x* pertenece al conjunto $\tilde{\boldsymbol{A}}$**.**

Para nuestros propósitos, nos restringimos a conjuntos difusos en la recta real R. Una función de pertenencia puede ser triangular, trapezoidal, sigmoidal, etc.

Definición 2. El conjunto difuso $\tilde{A}$ en R *es* normal si

$$\boldsymbol{max_x}\, \mu_{\tilde{A}}(x) = 1$$

Definición 3. El conjunto difuso $\tilde{A}$ en R *es* convexo si y solamente si la función de pertenencia de $\tilde{A}$ satisface la inecuación.

$$\mu_{\tilde{A}}[\beta x_1 + (1-\beta)x_2] \geq min[\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\,]$$

$$\forall x_1, x_2 \;\in R, \quad \beta \epsilon [0,1]$$

Definición 4. Un número difuso es un conjunto difuso normal y convexo de *R*

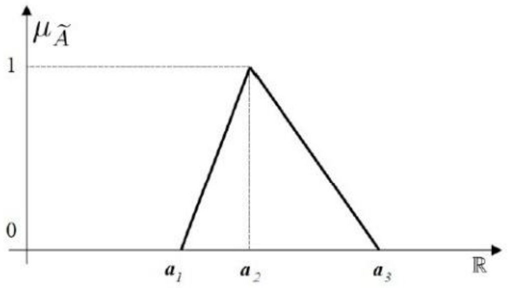

Figura 2: Número difuso triangular

Definición 5. Un número difuso triangular (NDT) (figura 2) puede ser definido como $\tilde{A} = (a_1, a_2, a_3)$. La función de pertenencia $\mu_{\tilde{A}}(x)$ de $\tilde{A}$ está dada por:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & si\; x \geq a_3\;, x \leq a_1 \\ \frac{x-a_1}{a_2-a_1}, & si\; a_1 < \; x \leq a_2 \\ \frac{x-a_1}{a_2-a_1}, & si\; a_2 \leq \; x < a_3 \end{cases} \tag{1}$$

Definición 6. Sea $\tilde{A} = (a_1, a_2, a_3)$ un conjunto difuso y un número real $\alpha \in [0,1]$. El conjunto clásico

$$A_\alpha = \{x : \mu_{\tilde{A}}(x) \geq \alpha, x \in \mathbb{R}\}$$

Es llamado $\alpha - corte\ de\ \tilde{A}$.

Al concepto de $\alpha - corte$ también se le llama conjunto de nivel (umbral) α. Este concepto permite un enfoque muy interesante de la teoría de conjuntos difusos, ya que la familia formada por los $\alpha - cortes$ contiene toda la información sobre el conjunto difuso.
El $\alpha - corte$ es quizás el concepto más importante de los números difusos, porque mediante el ajuste del valor, se puede determinar el rango o conjunto de valores que satisfacen un determinado grado de pertenencia o compatibilidad (presunción, certeza, son otras expresiones utilizadas), o expresado de otra manera el nivel de satisfacción, precisión del resultado o robustez del modelo.
Si consideramos el conjunto difuso con función de pertenencia de tipo triangular, $A = (a_1, a_2, a_3)$ de la figura 3, entonces:

$$A_\alpha = [a_1 + \alpha(a_2 - a_1), a_3 - \alpha(a_3 - a_2)]$$

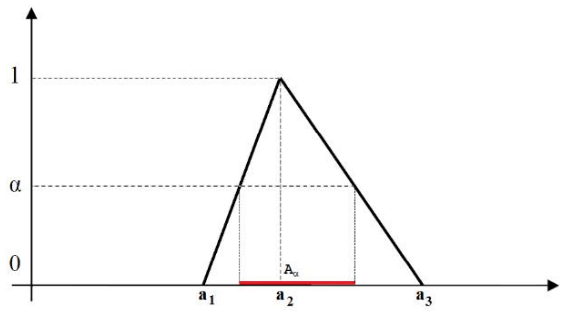

Figura 3: Intervalo correspondiente a un $\alpha - corte$ en un número difuso triangular

Distribuciones de Posibilidad

La ambigüedad se puede representar con distribuciones de posibilidad [16]. Estas distribuciones permiten formalizar de manera muy fidedigna gran cantidad de situaciones en las que se estiman magnitudes localizadas en el futuro. La medida de posibilidad de un evento puede ser interpretada como el grado de posibilidad de su ocurrencia en virtud de la distribución de posibilidad. Entre los diversos tipos de distribuciones, la triangular y la trapezoidal son los más comunes en la solución de problemas de programación matemática posibilista.

Formalmente, las distribuciones de posibilidad son números difusos, nos concentraremos sólo en NDT $\tilde{A} = (a_1, a_2, a_3)$, el cual está determinado por tres cantidades: a_2 es el valor con más posibilidad de ocurrencia, a_1 y a_3 son los valores límites inferior y superior permitidos, respectivamente (figura 3). Por ejemplo, estos valores límite pueden interpretarse como el más pesimista y el más optimista, en función del contexto.

Programación matemática difusa

La programación matemática difusa puede ser de gran ayuda para manejar situaciones en problemas de optimización que incluyen parámetros imprecisos [17].
Hay diferentes enfoques para la programación matemática difusa dependiendo del tipo de imprecisión en los parámetros del modelo a optimizar.
La programación difusa trata con parámetros vagos [18]. Por otra parte, la programación posibilista trata la ambigüedad, es decir, los datos disponibles se conocen exactamente aunque estos datos pueden variar dentro de un límite de tolerancia. En este caso, los parámetros se consideran como números difusos asociados con distribuciones de posibilidad ([14], [19]).

MODELO BAP DIFUSO CON IMPRECISIÓN EN LA LLEGADA DE LOS BARCOS

En esta sección, mostramos un modelo difuso para el BAP continuo y dinámico.
Se presenta la notación de los principales parámetros que se utilizarán en el modelo (figura 4).
L: longitud total del muelle del TMC.
H: horizonte de planificación.
Sea *V* el conjunto de barcos entrantes, los datos del problema para cada barco $i \in V$ están dados por:

- a_i: Tiempo de llegada al puerto.
- $\omega_i = m_i - a_i$: Tiempo de espera del barco desde que llega hasta que atraca.
- l_i: Longitud del barco.
- h_i: Tiempo de estancia del barco en el lugar de atraque (tiempo de servicio).

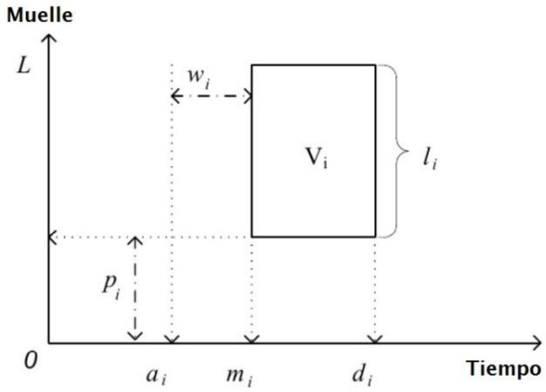

Figura 4: Representación de un barco de acuerdo a su posición y tiempo

Con estos datos, se debe decidir las variables.
m_i: Tiempo de atraque del barco.
p_i: Posición donde será atracado el barco.

El tiempo de partida del barco d_i dependerá de m_i y h_i. La posición de atraque p_i se determinará de acuerdo con la longitud de los barcos.
Tenemos en cuenta las siguientes suposiciones: Toda la información relativa a los barcos en espera se conoce de antemano, cada barco tiene un calado menor o igual que el muelle, el momento del atraque y desatraque no consume tiempo, está permitido el atraque simultáneo, no se considera distancia de seguridad entre los barcos.
El objetivo es distribuir todos los barcos de acuerdo a unas restricciones, minimizando el tiempo total de espera de los barcos.

$$T\omega = \sum_{i \in V} (m_i - a_i)$$

Con el fin de asignar un barco en el muelle, deben cumplirse las restricciones:
El tiempo de atraque debe ser al menos el mismo que el tiempo de llegada.

$$m_i \geq a_i.$$

Hay suficiente espacio contiguo en el muelle para atracar la embarcación.

$$p_i + l_i \leq L.$$

En este trabajo asumimos que el tiempo de llegada de un barco es ambiguo (impreciso), en el sentido de que los barcos pueden adelantarse o retrasarse hasta una cierta tolerancia permitida.

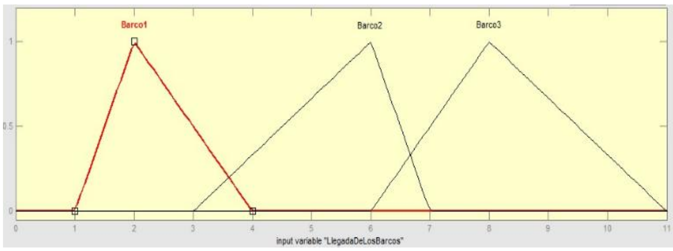

Figura 5. Distribuciones de posibilidad triangulares para la llegada de tres barcos

La llegada de cada barco se representa con una distribución de posibilidad triangular como en la figura 2, es decir, se considera que la llegada no será antes de a_1, ni después de a_2, la llegada con la máxima posibilidad es a_3.
En la figura 5, se muestra las distribuciones de posibilidad triangulares para la llegada de tres barcos. Por ejemplo, para el barco 1, la llegada no será antes de la 1 am, ni después de las 4 am. La llegada con la máxima posibilidad es a las 2 a.m.
Formalmente, consideramos que la imprecisión en la llegada de los barcos es un NDT $\tilde{a} = (a_1, a_2, a_3)$.
Con la posibilidad de adelanto o retraso en las llegadas y en base al modelo determinista [20], proponemos un modelo difuso para manejar la imprecisión.

$$mín\ \sum_{i \in V}(m_i - \tilde{a}_i) \qquad (2)$$

Sujeto a:

$$m_i \geq \tilde{a}_i \quad \forall i \in V \qquad (5)$$

$$p_i + l_i \ \leq L \quad \forall i \in V \qquad (6)$$

$$p_i + l_i \ \leq p_j + M\left(1 - z_{ij}^x\right) \quad \forall i, j \in V, i \neq j \qquad (7)$$

$$m_i + h_i \ \leq H \quad \forall i \in V \qquad (8)$$

$$m_j - (m_i + h_i\,) + M\left(1 - z_{ij}^y\right) \geq \widetilde{\boldsymbol{Pr}_{\boldsymbol{i}}} \quad \forall i, j \in V, i \neq j \qquad (9)$$

$$z_{ij}^x + z_{ji}^x + z_{ij}^y + z_{ji}^y \geq 1 \quad \forall i, j \in V, i \neq j \qquad (10)$$

$$z_{ij}^x, z_{ij}^y \in \{0,1\} \quad \forall i, j \in V, i \neq j \qquad (11)$$

Donde z_{ij}^{x} es una variable de decisión que indica si el barco i está localizado a la izquierda del barco j en el atraque ($z_{ij}^{x} = 1$), $z_{ij}^{y} = 1$ indica que el tiempo de atraque del barco i está antes que el del barco j. M es una constante entera grande.
La restricción (9), obliga que para un barco j que atraca después del barco i, su tiempo de atraque m_j incluya la precisión que se le puede tolerar al barco i. Esta precisión está representada por los números difusos $\widetilde{Pr}_i$ y es obtenida del número difuso $\tilde{a}_i$.
Este es un problema de programación lineal entero mixto difuso. Aunque los números difusos y números reales no son comparables, para propósitos ilustrativos, la comparación entre estos dos tipos de números ha sido presentada en el modelo previo. En la siguiente sección, el modelo es adaptado con el fin de realizar una adecuada comparación.

Aplicación al modelo BAP difuso

De acuerdo a la sección previa la llegada imprecisa de un barco se representa con el NDT $\tilde{a} = (a_1, a_2, a_3)$. Según la definición 6, su $\alpha - corte$ está dado por:

$$A_\alpha = [a_1 + \alpha(a_2 - a_1), a_3 - \alpha(a_3 - a_2)]$$

El $\alpha - corte$ representa el intervalo de tiempo que se permite en la llegada de un barco, para un grado de precisión. El tamaño de ese intervalo $\mathbf{Pr}(\alpha) = (1 - \alpha)(a_3 - a_1)$, es el tiempo que se tolera en la llegada del barco, el cual debe ser tomado en cuenta en el tiempo de atraque del barco que atracará después.
Se puede observar que para el valor α, el adelanto permitido es:
$\mathbf{ta}(\alpha) = (1 - \alpha)(a_2 - a_1)$

El retraso permitido es:
$\mathbf{tr}(\alpha) = (1 - \alpha)(a_3 - a_2)$

Y $\mathbf{Pr}(\alpha) = \mathbf{ta}(\propto) + \mathbf{tr}(\propto)$.

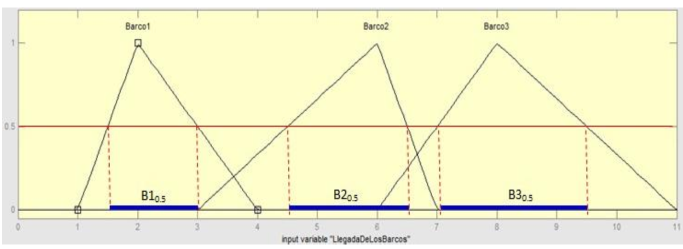

Figura 6: $\alpha - corte$ para $\alpha = 5$ para la llegada difusa de tres barcos

En la figura 6, se muestra los alfa cortes $B1_{0.5}$, $B2_{0.5}$ y $B3_{0.5}$ para la llegada de tres barcos, con un nivel de corte $\alpha = 0.5$.
Utilizando los alfa cortes como método de desfuzicación a la llegada difusa de los barcos, una solución al modelo BAP difuso se obtiene del siguiente modelo paramétrico MILP.

Entrada : V: Conjunto de barcos entrantes
Resultados: Planes de atraque para V con diferentes grados de precisión;
foreach $\alpha = \{0, 0.1, ..., 1\}$ **do**

/* adelanto permitido al barco i

$ta_i(\alpha) = (1-\alpha) * (a2_i - a1_i)$

/* retraso permitido al barco i

$tr_i(\alpha) = (1-\alpha) * (a3_i - a2_i)$

/* tiempo de tolerancia permitido a la llegada del barco i

$\mathbf{Pr}_i(\alpha) = ta_i(\alpha) + tr_i(\alpha) \quad \forall i \in V$

$$\text{mín} \sum_{i \in V} (m_i - (a_1 + \alpha * (a_2 - a_1)))$$

Sujeto a:

$$m_i \geq (a_1 + \alpha * (a_2 - a_1)) \quad \forall i \in V$$
$$p_i + l_i \leq L \quad \forall i \in V$$
$$p_i + l_i \leq p_j + M(1 - z_{ij}^x) \quad \forall i, j \in V,\ i \neq j$$
$$m_j - (m_i + h_i) + M(1 - z_{ij}^y) \geq \mathbf{Pr}_i(\alpha) \quad \forall i, j \in V,\ i \neq j$$
$$z_{ij}^x + z_{ji}^x + z_{ij}^y + z_{ji}^y \geq 1 \quad \forall i, j \in V,\ i \neq j$$
$$z_{ij}^x, z_{ij}^y \in \{0, 1\} \quad \forall i, j \in V,\ i \neq j$$

Figura 7: Modelo Paramétrico del modelo difuso

El horizonte de planificación está dado por

$$H = \sum_{i \in V} (h_i) + max\{a3_i, i \in V\}$$

En el modelo paramétrico, el valor de α es el grado o porcentaje de precisión permitido en el tiempo de llegada de los barcos. Para cada $\alpha \in$ *[0; 1*], y para cada barco *i*, se calcula los tiempos de tolerancia permitidos $\boldsymbol{Pr}$.
La solución del modelo proporciona planes de atraque que soporten un grado de precisión indicado por el tomador de decisiones. Por ejemplo, para un tiempo de llegada difuso $(a_1, a_2, a_3) = (22, 32, 47)$. Para $\alpha = 0.6$, el adelanto permitido es

$\mathbf{ta}(0.6) = (1 - 0.6)(32 - 22) = 4$; el retraso permitido es $\mathbf{tr}(0.6) = (1 - 0.6)(47 - 32) = 6$ y la tolerancia permitida es $\mathbf{Pr}(0.6) = 10$.
Cuanto menor sea el valor de α, mayor será el tamaño de la tolerancia permitida a cada barco. Por lo tanto, el plan de atraque incrementa su capacidad para soportar adelantos y retrasos más grandes, pero la función objetivo que minimiza el tiempo de espera para atracar y el menor tiempo de llegada posible del intervalo A_α se incrementa.

EVALUACIÓN

Los experimentos fueron realizados en 50 instancias. Estas consisten de 8 barcos con una distribución uniforme en los tiempos de llegada con retrasos.
La longitud del muelle es L=700. El algoritmo ha sido codificado y resuelto en forma óptima en CPLEX. Las instancias fueron resueltas en una computadora personal equipada con un Core (TM) i5 – 4210U CPU 2.4 Ghz con 8.00 Gb RAM.
Los experimentos se llevaron a cabo con un "*timeout*" (tiempo de cómputo máximo) de 60 minutos.
Los parámetros para los barcos que se utilizarán en esta sección son:
m1: tiempo de a*traque mínimo permitido*
m2: tiempo de atraque óptimo
m3: tiempo de atraque máximo permitido;
ta: tiempo máximo de adelanto permitido,
tr : tiempo máximo de retraso permitido.
d1: tiempo de salida *mínimo permitido*
d2: tiempo de salida óptimo
d3: tiempo de salida máximo permitido;

Una instancia se muestra en la tabla 1.

Tabla 1. Ejemplo de una instancia de los barcos

Barcos	a1	a2	a3	h	l
V1	4	8	34	121	159
V2	0	15	36	231	150
V3	18	32	50	87	95
V4	9	40	46	248	63
V5	32	52	72	213	219
V6	55	68	86	496	274
V7	62	75	90	435	265
V8	45	86	87	146	94

Por ejemplo, el barco *V1* debe llegar a las 8 unidades de tiempo, pero se le permite adelantarse y retrasarse hasta 4 y 34 unidades de tiempo respectivamente.

El valor de α representa el grado de precisión permitido a los barcos. Por ejemplo, $\alpha = 1$, significa 100% de precisión; $\alpha = 0.6$, es el *60* % de precisión permitido.
Para cada una de las instancias y para cada uno de los barcos, considerando once grados de retrasos ($\alpha = \{1,0.9,...,0\}$), se generaron 11 planes de atraque.
Como un ejemplo ilustrativo, para la instancia de la tabla 1, tres planes de atraque diferentes se muestran en las tablas 2, 3 y 4. Estos planes fueron obtenidos con el modelo paramétrico, variando el valor de α ($a = \{1, 0.5, 0\}$).

Tabla 2. Plan de atraque para α = 1

Barcos	a1	a2	a3	ta	tr	m1	m2	m3	h	d1	d2	d3	l	p
V1	4	8	34	0	0	8	8	8	121	129	129	129	159	541
V2	0	15	36	0	0	15	15	15	231	246	246	246	150	391
V3	18	32	50	0	0	32	32	32	87	119	119	119	95	233
V4	9	40	46	0	0	40	40	40	248	288	288	288	63	328
V5	32	52	72	0	0	52	52	52	213	265	265	265	219	0
V6	55	68	86	0	0	246	246	246	496	742	742	742	274	426
V7	62	75	90	0	0	265	265	265	435	700	700	700	265	0
V8	45	86	87	0	0	119	119	119	146	265	265	265	94	219

Para α=*1*, en todos los barcos los adelantos y retrasos son $ta = 0$ y $tr = 0$ (no se permite adelantos y retrasos en los barcos).
En la mayoría de los casos, si un barco se atrasa en llegar respecto a su tiempo preciso de llegada este plan deja de ser válido. Por ejemplo, el barco *V3* tiene un tiempo de atraque $m2=32$ y tiempo de salida $d2=11$; si este barco se atrasa, el barco *V8* no puede atracar en su tiempo asignado $m2 = 119$, y el que le sigue, el barco *V7* tampoco puede atracar en su tiempo asignado $m2$ = 265. Para un número mayor de barcos (como es en la realidad), el retraso de los barcos complica aún más los planes de atraque.

Tabla 3. Plan de atraque para α = 0.5

Barcos	a1	a2	a3	ta	tr	m1	m2	m3	h	d1	d2	d3	l	p
V1	4	8	34	2	13	6	8	21	121	127	129	142	159	219
V2	0	15	36	8	11	7.5	15	26	231	239	246	257	150	392
V3	18	32	50	7	9	25	32	41	87	112	119	128	95	605
V4	9	40	46	16	3	25	40	43	248	273	288	291	63	542
V5	32	52	72	10	10	42	52	62	213	255	265	275	219	0
V6	55	68	86	7	9	256	263	272	496	753	759	768	274	265
V7	62	75	90	7	8	275	282	289	435	710	717	724	265	0
V8	45	86	87	21	1	128	149	149	146	274	295	295	94	606

En α*=0.5* (una precisión permitida del *50%*), por ejemplo, para el barco *V3*, el tiempo de atraque óptimo es *m2=32*, el adelanto permitido es *ta= 7*, el retraso permitido es *tr = 9*, es decir, el barco puede atracar en el intervalo de tiempo *[25, 41],* y puede salir en el intervalo de tiempo *[112, 128]*. Después del barco *V3*, el barco *V8* puede atracar en el tiempo *m2 = 128* con un adelanto permitido de *ta =21* y retraso permitido de *tr=0.5*

Tabla 4. Plan de atraque para $\alpha = 0$

Barcos	a1	a2	a3	ta	tr	m1	m2	m3	h	d1	d2	d3	l	p
V1	4	8	34	4	26	4	8	34	121	125	129	155	159	282
V2	0	15	36	15	21	0	15	36	231	231	246	267	150	441
V3	18	32	50	14	18	18	32	50	87	105	119	137	95	605
V4	9	40	46	31	6	9	40	46	248	257	288	294	63	0
V5	32	52	72	20	20	32	52	72	213	245	265	285	219	63
V6	55	68	86	13	18	267	280	298	496	763	776	794	274	328
V7	62	75	90	13	15	285	298	313	435	720	733	748	265	63
V8	45	86	87	41	1	137	178	179	146	283	324	325	94	606

En *a=0* (una precisión permitida del *0%*), los adelantos y retrasos son incrementados, por ejemplo, para el barco *V3*, el tiempo óptimo de atraque es *m2=32* (el mismo que para *a = 0:5*), pero el adelanto permitido es *ta=14* y el retraso permitido es *tr=18*. Por lo tanto, el intervalo de tiempo donde el barco puede atracar es [18, 50].

Por la forma como se ha construido el modelo, para cada valor de *a*, los adelantos y retrasos permitidos son proporcionales a su tiempo máximo de adelanto y de retraso. Para el caso de los de retraso, por ejemplo, el barco *V1* y el barco *V3*, pueden retrasarse hasta un máximo de *26* y *18* unidades de tiempo respectivamente (ver Tabla 4). Para un grado de retraso permitido de *a=0.5*, los retrasos para el barco *V1* y el barco *V3*, son *tr=13* y *tr=9* respectivamente (ver tabla 3).

CONCLUSIONES

El sistema de un TMC requiere herramientas que ayuden a los administradores en la toma de decisiones. El BAP es uno de los problemas más críticos y estudiados en los TMC. Muchas investigaciones se ha desarrollado sobre el BAP; sin embargo, la mayoría asume que la llegada de los barcos es determinista. Esto no es real, en la práctica pueden ocurrir adelantos o retrasos en las llegadas de los barcos. Por lo tanto, la adaptabilidad de un plan de atraque es importante para el rendimiento global del sistema en un TMC.

En este trabajo, se ha presentado un modelo MILP difuso para el BAP continuo y dinámico. En el modelo propuesto, se asumió que el tiempo de llegada de los barcos es impreciso, en el sentido de que los barcos pueden adelantarse o retrasarse hasta un

grado de tolerancia permitido. Esta imprecisión se representó mediante conjuntos difusos con función de pertenencia de forma triangular.
Se utilizó el método de $\alpha - cortes$ para transformarlo en un problema de programación lineal paramétrica.
El modelo propuesto fue codificado y resuelto en forma óptima con la herramienta de optimización CPLEX. El modelo ha sido evaluado por medio de 50 instancias de ocho barcos. Se utilizó ocho barcos con fines ilustrativos, pero el modelo se comporta de la misma manera para un número mayor de barcos.
Los resultados obtenidos mostraron que el procedimiento puede ayudar a los administradores de un TMC en la toma de decisiones, pues tiene a su disposición planes de atraque con diferentes grados de precisión y optimizados respecto al tiempo de espera, con la característica que, a más precisión en la llegada de un barco, el modelo le otorga un tiempo de atraque mas preciso.
Finalmente, como resultado de la investigación, se abren posibilidades para futuras investigaciones:
(i) Extender el modelo propuesto, para tratar con problemas de optimización que contemplen la imprecisión que aparece en las llegadas y el tiempo de servicio de los barcos.
ii) Usar metaheurísticas con optimización difusa con el fin de solucionar en forma más eficiente el BAP difuso.

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Estrategias de optimización para el desarrollo energético sostenible en Angola

Optimization strategies for sustainable energy development in Angola

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RESUMEN

En este artículo se presenta un modelo matemático para apoyar el proceso de toma de decisiones relacionadas con la planificación de la capacidad y el funcionamiento de los sistemas de energía eléctricos híbridos. El modelo fue desarrollado teniendo en cuenta los casos de estudios desarrollados en pequeños pueblos, en las zonas rurales en Angola. Dependiendo de las condiciones locales, el sistema considera los siguientes tipos de generación de energía: fotovoltaicos, energía eólica, térmica y mini hidroeléctricas. Se evaluó la capacidad de producción mínima en cada localidad y se evaluó también la curva de duración de carga. Para cada tipo de fuente, se estimaron los costos de construcción y operación. El problema de optimización se resuelve como un problema de programación lineal estocástica de dos etapas, en el que los niveles de la curva de duración de carga son tratados como variables aleatorias. El objetivo es presentar, analizar y seleccionar la combinación híbrida que minimiza el costo total de la energía, estrategias para la satisfacción de las necesidades energéticas.

Palabras clave: optimización; sistemas de energía híbridos; gestión de la energía.

ABSTRACT

This paper presents a mathematical model to support the decision making process related to the planning of capacity and operation of hybrid electrical energy systems. The model was developed considering case studies of small villages, in rural areas in Angola. Depending of the local conditions, the system considers the following types of energy generation: photovoltaic, wind energy, thermal and mini hydro. The minimum production capacity in each locality was assessed and the load duration curve was also evaluated. For each type of source, the costs of construction and operation were estimated. The optimization problem is solved as a two-stage stochastic linear programming problem, in which the levels of the load duration curve are treated as random variables. The goal is to select the hybrid combination that minimizes the total cost of energy. Strategies for the satisfaction of energy needs are presented and analysed.

Keywords: optimization; hybrid power systems; energy management.

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INTRODUCTION

After the civil war in Angola, which lasted about 30 years, the economic fabric of the country, as well as the energy sector, were practically destroyed. Angola is one of the largest oil producers in sub-Saharan Africa; however, access to electricity today remains unavailable to approximately 70% of the population [1]. The chronic energy shortage of rural populations, which occurs in many developing countries, has led to the degradation of forests due to the use of their resources as fuel.

On the other hand, in recent years, different means of production and energy storage of flexible use have been developed, using renewable energy systems[2,3,4]. These technologies represent a potential opportunity for developing countries[5], and particularly for Angola.

These systems are called autonomous hybrid systems since they operate without connection to a neighbour network and combine different components of energy production[6]. In their composition we can find production units, such as mini-hydro units, photovoltaic cells, wind turbines and thermal generators. Hybrid systems might include also components of energy storage, such as batteries, and hydrogen production facilities; and elements for the recovery of stored energy, such as fuel cells, generators and fuel gas facilities[7,8].

The aim of this work is to contribute to the development of procedures to assist in making decisions associated with the composition and operation of autonomous hybrid systems. Through the application of the developed model to three case studies, we also present results that allow illustrating the economic implications of the solutions that are technically feasible, thus contributing to further clarification of the decision makers involved in this matter.

Mathematical models for the characterization of autonomous hybrid systems are particularly important under the conditions that characterize the situation in Angola and in other developing countries, particularly the climatic conditions, the low level of technological development and the lack of basic infrastructures. These systems offer a reasonable compromise between the investment costs and the operating costs[9,10] and on many circumstances, particularly in small villages in rural Angola, are the most viable alternative to meet the energy needs of the populations[11,12].

In order to analyze the energy needs of small villages, studies were developed in the municipal headquarters of Muxiluando and Ambriz, Bengo province, and in the municipal headquarters of Tombwa, province of Namibe. The objective is to size the components of energy production of autonomous hybrid systems considering photovoltaic, thermal, mini-hydro and wind alternatives, that minimize the investment and operating costs, taking into account the local needs, the technological environment and the weather and water conditions. The choice of the localities accounted for the proximity of small waterfalls that will serve for the construction of mini-hydro and/or localities with average wind speed exceeding 5 m/s.

Currently the studied localities are supplied by electricity locally thermal generated. The solutions studied aim to meet the current demand, and adapt the technologies and their economic, social and environmental impacts for sustainable development. Accordingly, we also consider the possibility of the growing of energy needs, thus resulting in the need of using optimization tools that take into account uncertainty.

There are several alternatives to formulate and solve the problem used to define the optimum composition of the hybrid system. These problems differ essentially with regard to the consideration, or not, of uncertainty and the consideration of one or more objectives. The simplest models are usually based on the minimization of the cost and do not consider the uncertainty[13,14]. The multi-objective models are used when it is desired to complement the economic analysis with the environmental impact[15]. A stochastic approach is generally used to introduce the effect of the uncertainty in the analysis[16]. If the uncertainty is related to the future demand it is obtained, generally, a stochastic programming problem in two stages with recourse[17,18].

Hence, in this study, the formulated optimization problem is a two-stage stochastic linear programming problem on the generation capacity and the power requirements of the different sources, which objective criterion is the equivalent investment costs plus the operating costs, subjected to constraints on the capacity of the power plants and on the available power, taking into account the uncertainty associated with the temporal evolution of the energy needs. The formulation presented also considers the possibility of energy transfer between nearby localities.

To emphasize the importance of combining different forms of energy as well as of the uncertainty in the optimization problem, are also analyzed solutions in which only one type of generation is considered and solutions without uncertainty for different scenarios of future needs.
This article is organized as follows. The next section presents data for the case study of Ambriz, related to the energy needs and the energy sources considered. Specifically, it is presented the data related with the investment and operating costs and the scenarios of energy needs. In the third section it is presented the formulation of the linear programming problem used for optimizing the hybrid system, considering the situation of deterministic demand. Based on this problem it is then defined the stochastic programming problem that incorporates the uncertainty related with the future evolution of the energy needs. The results of this study are then presented in the following section. The last section summarizes the main conclusions of this work.

Capacity planning

The decisions concerning the capacity planning of electricity generation have to account the current needs and constraints, but also their future developments, generally in a long time horizon with high uncertainty.

There are several types of uncertainty influencing the optimal decisions that must be taken in the first step, as for example the technological development of production systems and their impact on investment and operating costs, changes in demand and economic development, and the capability to promote and implement changes if necessary.

Therefore, in light of future developments, solutions that do not take into account potential risks of transforming decisions initially considered optimal in decisions with unnecessary costs for taxpayers, may have negative implications on the economic development and welfare of the population. These factors amplify the importance of developing strategies to minimize the risk with respect to a variety of future events in which the costs are explicitly considered.

The results presented in this study were supported by three real-world case studies involving the villages of Ambriz, Muxiluando and Tombwa. Yet, in this paper, the case of Ambriz is treated with more detail.

In the first phase of this study were assessed the minimum production capacity and the local energy needs. Following the guidelines in[19], the evaluation was performed by considering the consumption of households, and public and social services. A consumption of 13.2 MWh/day and a necessary power of 2.1 MW were estimated. Taking also in account cultural habits, the load duration curve (LDC) was constructed. The daily consumption was divided in three periods of different rates of consumption, leading to the LDC that is represented graphically in figure 1.

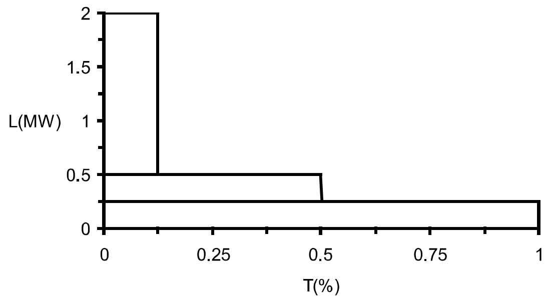

Figure 1. Distribution of the energy

The distribution of the energy needs for a daily cycle, for the current levels of consumption where the vector of average power levels is

$$L = \{0.25, 0.5, 2.0\} \text{ MW} \quad (1)$$

and the vector of time intervals, given as a percentage of the day in which the loads are exceeded, is

$$T = \{100, 50, 12.5\} \quad (2)$$

The next step was to decide the main alternatives for energy generation. Due to the proximity of water falls and a weak wind speed, we consider three different alternatives for energy production to be implemented together or separately: thermal, hydro and photovoltaic production.

For each alternative source of energy generation, costs of construction and operation were estimated. The cost structure was decomposed considering fixed and variable components and, taking into account the service life of the equipment and the opportunity cost of capital, the equivalent annual cost of each alternative was evaluated.

With regard to thermal production (equipment and installation), generators of 600 kW are available. Each generator has a cost of about 285 kUSD and a useful life of 10 years. In terms of hydro production, taking into account the flow rate and the height of fall of water, it is considered the construction of a small dam with a capacity of 7 MW. The estimated cost for this project is 23415 kUSD and the lifetime is considered infinite (i.e. more than 100 years).

The optimal strategy for the satisfaction of energy needs is determined solving an optimization problem, which goal is to select the hybrid combination of energy sources that minimizes the total cost of energy generation. To formulate this problem, equivalent investment costs in the same time base are evaluated. These costs represent an equivalent rent that tax payers would have to pay to an independent investor who decided to build and rent the infrastructure. This rent is equivalent to a fixed cost to be added to the fixed costs related to operating the system.

To evaluate the equivalent annual cost, we assume that the present value of the rents to pay the investor equals the value of the investment[20]. Accordingly, the discount rate is the opportunity cost of capital for investments of equal risk level. Since the rents would be guaranteed by the Angolan state, the appropriate cost of capital will be approximately 3% per year, which corresponds to the yield on long-term bonds issued in dollars by the Angolan state. Accordingly, the equivalent annual rent or equivalent cost is 33.411 kUSD for thermal production, 89.935 kUSD for photovoltaic generation, and 702.45 kUSD for hydro production.

To evaluate the remaining fixed costs, we consider essentially maintenance costs, direct labour costs and transportation costs. Based on the needs for each type of generation, we obtained the following results: 20.1 kUSD a year per generator for thermal production; 53.03 kUSD a year per unit of 300 kW for photovoltaic production; and 1078.7 kUSD a year for hydro production.

Adding the equivalent cost to the fixed operating costs leads to the annual fixed cost that covers the financial and operational components. Dividing this value by 365 days and the production capacity of each unit, a (daily) equivalent acquisition cost per unit of capacity is obtained. Using the indices 1, 2 and 3 to designate respectively the means of production thermal, hydro and photovoltaic, the vector of equivalent acquisition costs is

$$c = \{0.244, 0.697, 1.306\} \ \frac{\$}{kW\ day} \qquad (3)$$

The variable costs are the opportunity costs of the resources used to produce an additional unit of energy. In our case, we have to consider only the costs of production of thermal energy. To assess these costs we consider the calorific value of the fuel as 42.7 MJ/kg and the efficiency of the generators as 30%. If the opportunity cost of fuel is 1 $/kg, since this cost is zero for the other means of production, we obtain the following vector of variable costs:

$$f = \{0.281, 0, 0\} \ \frac{\$}{kW\ h} \qquad (4)$$

The stochastic parameters considered in the model correspond to the uncertainty of the evolution of energy needs. In this study we consider three possibilities for each level of the LDC that correspond to the current energy needs, and their duplication and triplication. We assume the three levels of power perfectly correlated and consider two cases for the probabilities of occurrence; firstly that they are equal and secondly the following values: 1/2, 1/3 and 1/6 respectively for the current needs, their duplication and their triplication. These scenarios are consistent with the forecasts of growing energy needs worldwide[21] and with an expected increase, above average, in energy needs in Angola, in the coming decades.

Modelling approaches

Before the formulation of the optimization problem, let's consider the assumption of no combination of the production systems. In this case, the most economically advantageous solution can be evaluated directly by comparing the total cost for the different alternatives with or without the use of excess capacity.

If it is not possible to take advantage of excess capacity, the available alternatives correspond to acquire the minimum required capacity and the best solution depends on maximum power and daily energy consumed. A photovoltaic power plant may be the best alternative for the actual scenario, with a daily cost of 2742 USD, against the cost for thermal production of 4382 USD and the cost of hydro production of 4880 USD. However, if the consumption duplicates, the mini-hydro plant should be the best solution keeping the daily cost equal to 4880 USD.

To account for the benefits of overcapacity, particularly in water production, we have to introduce a price of energy transfer to other locations. In this case, the maximum allowance is the fuel savings induced in neighbouring towns, which only alternative is the thermal production. However, to evaluate a sale price to other localities, we should consider that not all the energy transferred is received by potential customers. Additionally, the means of production themselves are already installed, so it will be convenient to provide incentives to not use them. So, in order to determine the maximum selling price P_E of the kWh produced to other localities, we introduce two additional parameters: the transfer efficiency and the incentive factor of not using thermal production. Considering both of these factors equal to 0.75, we get $P_E = 0.158$ \$/kWh.

Now, to evaluate the possibility of combining two or more sources of energy we consider also the possibility of receiving energy from other systems, being P_D the minimum acquisition unit price of thermal energy from nearby localities. Considering the same transfer efficiency and assuming a profit margin of the producer equal to 0.25 we get $P_D = 0.469$ \$/kWh.

Deterministic demand

The LDC of the figure 1 shows the distribution of the energy needs for a daily cycle, considering the current levels of consumption. In practice, the consumption and its daily distribution are time varying and are subject to uncertainty. Then, the optimization problem is solved as a two-stage stochastic linear programming problem, in which the levels of the load duration curve are treated as random variables.

An approximate solution can be obtained by ignoring the uncertainty on the fluctuations of the LDC levels, and using the expected values for a distant horizon. In the formulation of this problem are involved the decision variables, constraints and the objective of the problem with uncertainty. Additionally, the solution of this problem is a benchmark from which we can identify the effect of the uncertainty in the decision process.

The objective of the problem is to define and scale the means of production so as to minimize the total daily cost. Two types of decision variables are identified: in the first step it is necessary to define the generation capacity; subsequently it will be necessary to define the operational plan of energy production to match supply and demand.

In this context, the decision variables of the first phase x_j are the generation capacity (in kW) of type *j*. The decision variables of the second phase y_{ij} represent the power requirements of the segment *i* served by generator *j*. As the capacity of the plants varies only by constant increments, we also introduce auxiliary variables z_j as the number of production units of the central production of type *j*. To perform the operational plan (i.e. the allocation of powers of generators to the required power levels) the segments of power w_i are still defined, as well as the corresponding time intervals β_i, on the adopted daily basis. Then, the optimization problem can be formulated as

$$Min\ C_T = \sum_j c_j x_j + \sum_i \sum_j f_j \beta_i y_{ij} + P_D \sum_i \beta_i d_i - \sum_j \left(P_E - f_j\right) E_j \quad (5)$$

s.t.

$$\sum_j x_j \geq Q_d$$

$$\sum_j x_j \leq Q_u$$

$$\sum_i \beta_i y_{ij} + E_j - \beta_1 x_j \leq 0 \quad \forall j \in J$$

$$\sum_i y_{ij} + q_j - x_j = 0 \quad \forall j \in J$$

$$\sum_j y_{ij} + d_i = w_i \quad \forall i \in I$$

$$x_j - Q_j z_j = 0 \quad \forall j \in J$$

$$x_j, y_{ij}, q_j, d_i, E_j \geq 0,\ z_j \in \mathbb{Z} \quad \forall i \in I, \forall j \in J$$

The terms of the objective function represent respectively the total fixed costs, the operating costs, the acquisition costs of energy from the outside, and the contribution margin of sales.

The parameters Q_d and Q_u are the lower and upper limits of the total power to install. The third set of constraints results from the balance between the energy available from the system, the power supplied to the segments of power, and the energy available to transfer to the exterior E_j. The fourth set of constraints limits the available power of the medium *j* at its nominal power. The fifth set of constraints requires that each segment of power receives the necessary energy, using if necessary the external source d_i. The sixth set restricts the capacity to be installed by constant increments, that is $Q=(600,7000,300)$ kW for the case of Ambriz, being Q_j the power of the unit of source *j*.

Uncertain demand

The linear programming model presented above calculates the optimal investment plan and the operational strategies for the deterministic problem of capacity setup. With perfect information, this solution represents a lower bound of the present value of the costs for a given scenario. However, due to the difficulty in predicting the future evolution of the factors mentioned in the previous sections, it can not be assumed that an optimized strategy for a given scenario is optimal for a set of realizations of those factors.

Omitting the uncertainty of the random factors can lead to solutions of limited utility[16]. Moreover, the stochastic programming approach provides optimized solutions that take into account the possibilities of realization of the random variables, in particular considering specific strategies to adapt to these realizations.

The stochastic problem that is obtained from the deterministic problem, by incorporating the uncertainty related with the future evolution of the energy needs, is a decision problem in two stages, where in the first are considered the planning decisions of the system, and in the second are considered the operational decisions.

The formulation of two-stage stochastic programming problems can be attributed to[22,23,24]. The problem formulated in this study can be written in the typical form of these problems, so assuming the following form

$$\begin{aligned} &\text{Min}\left\{c^T x + E_\omega\left[Q(x,\omega)\right]\right\} \\ &s.t. \quad Ax \le b \\ &\qquad\ \ x \in R_+^{n_x} \end{aligned} \tag{6a}$$

with

$$\begin{aligned} &Q(x,\omega) = \text{Min}\ q_\omega^T\, y \\ &s.t. \quad T_\omega\, x + W y \ge h_\omega \\ &\qquad\ \ y \in R_+^{n_y} \end{aligned} \tag{6b}$$

where *x* and *y* are the first-stage and the second-stage decision vectors, respectively, $X \subset R_+^{n_x}$, $Y \subset R_+^{n_y}$, and n_x and n_y represent the number of components of these vectors. The goal is to determine the optimal values of the first-stage variables that minimize the expected total cost, considering all-possible actions that can be taken after full information is available.

The vectors c and q stand for the specific costs associated respectively to the first-stage and second-stage decisions. The random variable
second-stage decision problem and is defined on the probability space (Ω, F, P), where F is a complete and right continuous filtration and P is a probability measure on Ω.

Although all the parameters in the second-stage problem can be random, models with all this flexibility are rare[25]. Commonly, the recourse matrix W is assumed constant as well as the second-stage costs q. It is also assumed that all stochastic components are known functions of a vector $\xi(\omega)$ of random factors and a known joint probability distribution. The operator E_ω is the expectation under the probability measure P.

When uncertainty is characterized by multiple correlated parameters, it is usually impractical to solve the problem by analytical manipulation of the respective multivariate probability distribution. Under these circumstances, the first step in solving stochastic programming problems is to assemble finite distributions with identical properties (e.g. expectation and covariance matrix). Then, the random vector $\xi(\omega)$ is discretized in a finite collection Ξ of representative values, and a probability distribution P on Ξ is defined such that the following identity holds:

$$\bar{Q}(x) = \sum_{\xi \in \Xi} Q(x, \xi) P(\xi) \tag{7}$$

With a discrete distribution of $\xi(\omega)$ a deterministic equivalent linear programming (DELP) problem can be formulated[26]. The expected value in the objective function of the first-stage problem is replaced by $\bar{Q}(x)$ and the restrictions of the second-stage problem originate additional restrictions, and additional variables, associated to each one of the realizations of the random factors. If the random vector is discretized by n_R representative values (i.e. scenarios), with corresponding probabilities p_k $(k = 1, 2, \ldots, n_R)$ the DELP problem will be expressed as

$$\begin{aligned} &\text{Min } E[c_T] = c^T x + \sum_k p_k q_k^T y_k \\ &S.t. \quad Ax \le b \\ &\qquad T_k x + W y_k \ge h_k, \quad \forall k \in K \\ &\qquad x, y_k \ge 0, \qquad x \in X, y_k \in Y, \forall k \in K \end{aligned} \tag{8}$$

Since our example only considers 3 scenarios for the realization of uncertain parameters, solutions with uncertainty presented are evaluated from this problem. The equivalence between this problem and the problem defined by expressions (6a) and (6b) is also demonstrated in[27], which emphasize that the use of this formulation is limited to cases where the number scenarios is not very high.

RESULTS

Village of Ambriz

1. Deterministic demand

In programming problems with uncertainty and two-stages with recourse, simplified formulations can be adopted which result from approaches that do not take into account the specific response to the realization of each of the scenarios considered. The simplest approach is to tackle the so-called underlying deterministic problem using the expected values of uncertain parameters. Another common approach is the wait-and-see approach. This approach consists in evaluating the solution for each scenario and to adopt one of them, or a combination of the solution values of the various scenarios, such as the average value.

When disregarding the use of surplus capacity – available for example for the mini-hydro choice – the optimum solution to meet the requirements, if maintained at the current level, would be: $x = [1200, 0, 900]$ kW with $C_T = 2396$ USD/day. These results show that under these conditions the dam would not be economically feasible. The best solution would be a combination of thermal and photovoltaic production. In this case the first two levels of the LDC would be met by the photovoltaic production, $y_{13} = 250$ kW and $y_{23} = 250$ kW, and the third one by both types of production, thermal $y_{31} = 1100$ kW and photovoltaic $y_{33} = 400$ kW. Comparing the best total daily costs between this case and the cases in which only one type of generation at each time can be used, we find the combined solution would cost about 13% lower than the minimum obtained in those cases of no combined solution.

If we assume that the energy needs will double for sure, the best solution would be: $x = [3000, 0, 1200]$ kW with $C_T = 4660$ USD/day. This solution also considers as more advantageous the combination of thermal with photovoltaic generation. However, in these conditions, the total daily cost for the solution corresponding to the mini-hydro only generation is slightly higher (about 5%). For the hypothesis of tripling the energy needs, the most economically advantageous solution corresponds to exclusively using the mini-hydro, with a total cost of 4880 USD/day.

We now consider the situation in which exchanges with the outside, purchase and sale, are possible. In this case, since excess capacity yields financial benefits, the solution depends on the upper limit imposed on the total capacity to install. If the upper limit is equal to the capacity of the dam, and it is possible to make full use of the surplus energy that can be produced, the optimal solution in any of the scenarios considered is the construction of the dam $x = [0, 7000, 0]$ kW. Accordingly, for example at the current conditions, one would obtain an excess of energy of 154500 kWh/day, with a potential benefit of 24400 USD/day. In this case, instead of a cost we would have a benefit for Ambriz of 19520 USD/day.

2. Stochastic demand

The consideration of uncertainty in the problem aims to add information to the decision process, which helps to identify the support conditions of each of the alternatives. Thus, we begin by examining the case where the three scenarios are equally likely, and without use of surplus capacity. In this case, despite the average needs coincide with the values of the average scenario, i.e. 27 MWh/day, the optimal solution is the construction of the dam, with a total daily cost of 4880 USD/day.
However, continuing to assume that there is no use of surplus capacity, if the expected long-term needs are sufficiently lower than the average scenario, the optimal solution involves using a combination of thermal and photovoltaic energy. For example, considering the probabilities of 1/2, 1/3 and 1/6 for scenarios where energy needs remain, double and triple, respectively, the expected long-term needs become 22.5 MWh/day and the optimal solution becomes the following: $x = [4800, 0, 1200]$ kW with $E[C_T] = 4664$ USD/day. In operational terms the energy needs are met using first the photovoltaic source and when this is not enough comes into operation the thermal production.

On the other hand, considering the possibility of using part of the surplus capacity, if the expected long-term needs are of 22.5 MWh/day, the optimal solution is back to the construction of the dam. In this case, the excess energy will be of 154500, 141000 and 127500 kWh/day, respectively for each scenario; and if we consider a utilization of only 5% of the excess energy, we obtain an expected benefit of 1150 USD/day, this fashion reducing the present value of the expected total daily cost to 3730 USD/day.

3. Villages of Muxiluando and Tombwa

Relatively to the cases of Muxiluando and Tombwa, the current energy needs were estimated as being 7.4 and 33.8 MWh, respectively, with daily distributions identical to the distribution of Ambriz. The hydro and wind energy alternatives are not available for the village of Muxiluando. In the village of Tombwa there is no possibility of hydroelectric power but there are conditions of exploitation of wind energy, which has equivalent acquisition costs identical to the costs of hydropower.

Due to the proximity between the villages of Ambriz and Muxiluando, the results presented for Ambriz allow to conclude that the joint needs of the two villages contribute to strengthening the conditions for economic viability of the implementation of the mini-hydro in Ambriz.

The village of Tombwa is far from the other two villages. In this case, it would be necessary to consider a solution based on wind energy, and for technical reasons it could include the thermal and photovoltaic productions.

CONCLUSIONS

The motivation for this work arose from the need to find sustainable solutions that contribute to minimizing the cost of energy needs of rural populations in Angola. As a reflection of this problem it was found that, despite being a major oil producer, per capita consumption and energy intensity in Angola are quite low. This situation results from the long period of war, lack of investment in the power sector and the non-use of renewable energy.

This paper presents a study on the optimization of hybrid systems for generating electrical energy, including the following energy sources: photovoltaic, wind, hydro and thermal. The optimization problem is formulated as a two-stage stochastic linear programming problem on the generation capacity and the power requirements of the different sources.

The conclusions of this study are supported using real data from three case studies that were developed in the villages of Ambriz, Muxiluando and Tombwa. The aim in each case was to find the best solution for electricity generation seeking the exploitation of natural resources.

One should note that the use of natural resources, namely hydropower, has economic advantages even when energy needs are lower than the potential of generation available. An important contribution to support this conclusion comes from the consideration of uncertainty regarding future energy needs. We also found that the possibility of energy transfer between villages also reinforces the advantages of this solution.

AWARDS

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IV

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Empirical analysis of a pig supply chain by a two-stage stochastic program

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RESUMEN

Este trabajo presenta la formulación y el análisis empírico de un modelo estocástico de dos etapas con el objetivo de optimizar el sistema de producción de cerdo según la nueva estructura de cadena de suministro que se observa en los últimos años. El modelo está diseñado desde el punto de vista práctico y proporciona la planificación de la producción óptima: la programación de transportes entre granjas, ocupación de instalaciones y camiones necesarios. Los parámetros del modelo representan una compañía de carne de cerdo de tamaño medio en España. Las variables enteras del modelo hacen difícil el encontrar una solución óptima exacta con un tiempo computacional razonable. Teniendo esto en cuenta, se realizó un análisis empírico para reducir el tiempo computacional que mantiene su buena situación suficientes resultados. Relajando la integralidad y modificando el horizonte de tiempo que afectan directamente el tiempo de resolución fueron algunas alternativas probadas. Demostramos que podemos lograr buenos resultados con tiempos computacionales razonables y con utilidad práctica. Son presentadas como una conclusión, alternativas adicionales y futuras formas de investigación colaborativa.

Palabras clave: programación estocástica en dos etapas; producción porcina; gestión de la cadena de suministro.

ABSTRACT

This talk presents the formulation and an empirical analysis of a two-stage stochastic model with the aim to optimize the pig production system according to the new supply chain structure arising in the latest years. The model is intended for practical purpose and provides the optimal production planning: the scheduling of transfers between farms, occupancy of facilities and trucks needed. Model parameters represent an average size pork company in Spain. Integer variables of the model make hard to find an exact optimal solution in a reasonable computational time. Taking this into consideration, an empirical analysis to reduce computational time maintaining good enough results was done. Relaxing the integrality and modifying the time horizon affecting directly the solving time were tested. We demonstrate we can achieve good enough results with reasonable computational times and so, practical use is feasible. As a conclusion, additional alternatives and future ways of collaborative research are presented.

Keywords: two-stage stochastic program; pig production; supply chain management.

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INTRODUCTION

During past decades, the pig industry has greatly evolved in western countries and especially in Europe [9,10,16]. As a result, the profile of the typical farm is changing from a family-based, small-scale and independent firm to one in which larger firms are more tightly aligned along the pig production and distribution processes and integrates their operations into a Supply Chain structure [17, 18]. Typically, in a Pig Supply Chain (PSC) the production process is structured through three stages or phases (Figure 1) encompassing different agents or farmers. Briefly, the first phase focuses on producing the piglets, the second phase focuses on rearing the piglets and the third and last phase focuses on fattening the pigs and delivering them to the abattoir. Piglets stay in all of these phases for a certain number of weeks in order to ensure the correct growth, weight by stage, health and welfare conditions. For each of these phases, a set of specialized farms are involved. Each one has their own characteristics, facilities and location. Therefore, transportation between phases is necessary making production planning more complex.
PSC's have competitive and health control advantages [5,10] because of the coordination within the chain, which helps reduce diseases' risk, uncertainty and creates value [12,17]. In this context, the planning of piglet production, the control of animals stock over time and the scheduling of transfers among farms deserve the attention of chain managers who have to allocate resources and time to solve these questions properly. In view of the complexity of the problem and the relatively quite recent novelty, most companies perform these tasks by hand or with rudimentary support that are not capable of representing the whole PSC or accounting for transport cost considerations. On the other hand, a lack of suitable models formulated for this kind of problems is detected [10,16] representing an opportunity for operational research to contribute [12,14].

The aim of this work is to explore computational results with regard to a practical use for PSC managers extending the deterministic model of Nadal-Roig and Plà [8] to a two-stage stochastic model with pork sales prices as uncertain parameters. Given the presence of integer variables, it is expected to get troubles in the resolution of the model, so computational experiments leading to a compromise between good enough solutions and solving times are presented. This empirical analysis is aimed at supporting practical short-term decisions like the transfers between farms and stock's management.

MATERIAL AND METHODS

Two-stage stochastic linear models provide a suitable framework for modeling decision problems under uncertainty arising in several applications [2,19]. The proposed two-stage stochastic linear model is an extension of the deterministic linear model by Nadal-Roig and Plà [8]. The formulation follows the Deterministic Equivalent Model (DEM) proposed in Birge and Louveaux [2] and Escudero et al.

[3]. In our approach, the uncertain parameters are those related with future sale prices and the sows' fertility of sows. Uncertainty is represented in the model by a set of possible scenarios, S, with corresponding probabilities ps. In Figure 2a, the scenarios are shown independently. Solving the problem for each scenario would produce wrong solutions [2,4]. Thus, some new constraints have to be added to satisfy the so-called non-anticipativity principle [3]. This is, all the scenarios must have the same first stage variables. The flexibility of these models is related to their multiperiod nature, i.e. besides the first stage variables that represent decisions made in face of uncertainty, the model consider second stage decisions, i.e. recourse actions, which can be taken once a specific realization of the random parameters is observed. Hence, the decision at the first stage (St1) it is the same for all scenarios while the remaining decision variables are dependent of the corresponding scenario, $s \in S$. In our case, these variables they represent the total weekly inventory of piglets and age-ranked for all farms. Those decisions of particular interest; they determine the weekly transfers (origin and destination) of animals (piglets, pigs and sows) between farms and to the abattoir; and states the number of trips necessary to perform the transfers.

These Sow herd structures are is known from the beginning, making the production of piglets being steady over time and for all the scenarios depending on sows' fertility. Furthermore, this is in agreement with a targeted number of weakly farrowings and weanings usually imposed by the PSC managers [8,12,15].

Some sets and parameters have to be defined to formulate the model in detail:

Sets and indexes:

$s \in S$: a finite set of scenarios

$t \in T$: a finite set of weeks

$T_1 \subset T$: a subset of T corresponding to the periods of the first stage

$h \in H$: Farm in the pig production process with a total of $|H|$ farms. $H=B \cup R \cup F$ disjoint partition of farms, being B the sow farms, R the rearing farms and F the fattening farms.

$e \in E$: Growing stage with a total of $|E|$ growing stages where E represents the productive cycle for obtaining a commercial pig ready to be slaughtered. Each stage corresponds to one week in the production system.

$E=E_B \cup E_R \cup E_F$ disjoint partition of growing stages housed in different facilities, being E_B the lactation period (from the birth to the weaning of piglets), E_R the rearing period (from weaning to the beginning of the fattening) and E_F corresponding to the fattening period.

$S_a \in S$: physiological state with a total of $|S|$ states in the end of which sows are culled and sent to the abattoir.

$S_g \in S$: Farrowing state with a total of $|S|$ states in the end of which farrowing take place and piglets are born.

$w \in$ W: Marketing window in the fattening stage where the pigs can be transferred to the abattoir.
As a summary and for a better comprehension to the reader, Fig. 5 illustrates the sets defined previously and the relationship between them in the pig production system.

Parameters:
w_s: Weight of the scenario $s \in$ S
pp_{st}: expected pork value per kilo for the scenario $s \in$ S at week $t \in$ T
ps_{st}: expected culled sow value per kilo for the scenario $s \in$ S at week $t \in$ T
fs_{st}: averaged fertility of sows for the scenario $s \in$ S at week $t \in$ TIN_{he}: initial inventory of pigs of age e in the farm h $\in$ H
Jr_e: Final inventory of pigs of age e at rearing farms
Jf_e: Final inventory of pigs of age e at fattening farms
p_{ij}: transition probabilities of sows from i to j, with $i,j \in S$ in sow farm $b \in$B
K_h: capacity in number of sows if $h \in$B or pigs if $h \in$R$\cup$F.
LS_{bi}: litter size at farrowing in the $b \in B$ sow farm per reproductive cycle $i \in$S
$Cp_{t,e}$: unitary cost per week t$\in$T and stage e$\in E$ per piglet including feeding, medicines and associated costs.
Cs: unitary cost per week per sow including feeding.
Ct: unitary cost per kilometre of a truck.
$d(h,h^*)$: distance from farm h to another farm or to the abattoir, $h^* \in$H$\cup$\{a\}.
ka_h: capacity in number of animals a truck can transport from farm $h \in$ H – R to the abattoir.
kg_h: capacity in number of animals a truck can transport from farm $h \in$ H – F to another farm
D: minimum number of pigs that the abattoir can accept for a week t
$pp_{g,t,e}$: expected pork value per kilo for the scenario g $\in$ G at week t and stage $e \in E_F$
$ps_{g,t}$: expected culled sow value per kilo for the scenario g $\in$ G at week t

Decision variables
π_{bi}: steady state inventory of the total number of sows at physical state $i \in S$ in the sow farm $b \in$B
π_{ba}: available number of sows to take to the abattoir in the $b \in$B sow farm each week.
I_{ste}: total inventory of piglets of age $e \in$E at week t in the system for each scenario $s \in$S.
I_{sthe}: inventory of piglets of age $e \in E_h$ at week t on the farm h for each scenario $s \in$S
A_{sth}: inventory of oldest pigs (greatest age) allowed on farm $h \in$ B$\cup$R, at week t to be transferred to the next stage in the chain for each scenario $s \in$S
A_{stfe}: Inventory of oldest pigs at age $e \in E_F$ allowed on fattening farm $h \in$F, at week t to be transferred to the abattoir for each scenario $s \in$S

A_{stbr}: inventory of piglets at week t transferred from $b \in$B to $r \in$R farm for each scenario $s \in$S

A_{strf}: inventory of piglets at week t transferred from $r \in$R to $f \in$F farm for each scenario $s \in$S.

Nka_{sth}: number or trucks necessary at week t to transport animals leaving the farm h to the abattoir for each scenario $s \in$S

$Nkg_{sth_1h_2}$: number or trucks necessary at week t to transport piglets covering the path between farm h_1 and h_2 being $h_1 \in$B and $h_2 \in$R or $h_1 \in$R and $h_2 \in$F for each scenario $s \in$S

Structure of the objective function

The aim of the model is to maximize the expected profit over a set of scenarios, $s \in$ S, i.e. the expected income minus expected operational cost over a finite time horizon $t \in$ T (t in weeks) per scenario. The income is calculated according an average sale price per kilo of live-weight produced, ps_{st}, considering a distribution of weight and lean percentage for pigs produced [7, 9, 14]. Sows are culled after a number of farrowings, $\pi_{b,a}$, and sent to the abattoir. Culled sows are replaced by new ones referred as gilts.

Fattening farms cannot deliver pigs to the abattoir at any time. The deliveries are allowed during a marketing window time, once enough pigs in the farm have reached marketable weight. Hence, fattened pigs, A_{stfe}, of farm f, weighting W_e at the fattening stage e sent to the abattoir at week t are sold at a price of pp_{st} € per kg of live weight. However, penalties or bonus are applied depending on lean content and carcass weight. The marketing time window allows the manager to be flexible selling animals when necessary. Regarding operational cost, it is calculated from a weekly unitary cost applied to the inventory of animals and the transportation cost to transfer the animals to the next phase or to the abattoir are considered. The former, C_s, and Cp_e, is expressed in €/animal and summarizes animal feeding, doses of insemination, labor and veterinary expenses and the latter the transportation cost, Ct, expressed in €/km. The number of trips needed to transport the animals to the abattoir $Nka_{s,t,h}$ or between farms $Nkg_{s,t,h}$ being the distance $d(h,a)$, $d(b,r)$, $d(r,f)$ in km for the farms that transfer animals to the abattoir, from sow farms to rearing farms and from rearing farms to fattening farms respectively. Transportation is done by a third party under a global cost per truck based on the distance between farms or farm to abattoir to cover regardless the number of animals. The number of pig and the number of trucks to transport them are defined originally as integer variables. Therefore, the structure of the objective function is defined as follows:

$$\max \sum_{s \in S} w_s \cdot \sum_{t \in T-1} \left(\sum_{b \in B} W_{|E_F|} ps_{s,t} \pi_{b,a} + \sum_{f \in F} \sum_{e \in E_F} W_e pp_{s,t} A_{s,t,f,e} - \sum_{b \in B} \sum_{i \in Si} Cs \cdot \pi_{b,i} - \sum_{h \in H} \sum_{e \in E} Cp_e I_{s,t,h,e} \right) -$$

$$\sum_{s \in S} w_s \cdot \sum_{t \in T} \left(\sum_{h \in H} Nka_{s,t,h} Ct \cdot d(h,a) + \sum_{b \in B} Nkg_{s,t,b} Ct_t d(b,r) + \sum_{r \in R} Nkg_{s,t,r} Ct \cdot d(r,f) \right)$$

Constraints of the model

Capacity of facilities: All facilities have a limited capacity. For the sow farms (Eq. 1), the capacity K_b depends on the number of sows $\pi_{b,i}$ that can be housed while in rearing and fattening farms (Eq. 2), the capacity K_b depends on the maximum number of pigs that can be fed at a time. Unlike the sow farms, which are operating under a steady state, the inventory in rearing and fattening farms $I_{s,t,h,e}$ has to be considered per each group of scenario:

$$\sum_{i \in S_i} \pi_{b,i} \le K_b \qquad b \in B \qquad (1)$$

$$\sum_{e \in E} I_{s,t,h,e} \le K_h \qquad s \in S, t \in T, h \in H - B \qquad (2)$$

The capacity of each farm must be considered each week. The sow herd dynamics is modelled as a Markov Decision Process as described in Plà et al. [13]. For this reason, a set of constraints representing the steady state in each sow farm has to be considered:

$$\pi_{j,b} - \sum_{i \in S} p_{i,j} \pi_{b,i} = 0 \qquad j \in S, b \in B \qquad (3)$$

Initial inventory: All farms which is part of the supply chain must have an initial inventory $IN_{b,e}$ per farm h and stage e at the beginning of the planning horizon. This initial inventory affects the flow of animals along the chain in the succeeding weeks and over the time horizon period which is being considered:

$$I_{0,0,h,e} = IN_{h,e} \qquad e \in E, h \in H \qquad (4)$$

Growth of animals: Pigs, which are fed on farms, grow from one stage to the next week by week. We assume that all pigs grow are fed under the same regime and grow accordingly to their age. Therefore, the inventory I must reflect this changing situation week by week over the time horizon considered and has to be updated. These constraints can be stated for the whole system (Eq. 5) or for each phase of the supply chain (Eq. 7, 9 and 11). These constraints makes the piglets to stay in each phase (sow, rearing and fattening) the number of weeks defined by E_B, E_R and E_F respectively. No casualties are considered during the growing process. If there are any casualties, they are taken into account when animals are transferred to the following phase in the chain. The constraints can be relaxed by using "less or equal" constraints. Let's note Eq. 6, 8, 10 and 12 corresponds to the non-anticipativity constrains:

$I_{s,t,e+1} = I_{s,t-1,\mathrm{e}}$ $\quad e \in \mathrm{E} \backslash \{|\mathrm{E}|\}, t \in \mathrm{T} \backslash \{1\}, \mathrm{s} \in \mathrm{S}$ (5)

$I_{s,\mathrm{t},e} = I_{1,t,e}$ $\quad e \in \mathrm{E} \backslash \{|\mathrm{E}|\}, \mathrm{s} \in \mathrm{S}, t \in \mathrm{T}_1$ (6)

$I_{s,t,b,e+1} = I_{s,t-1,b,e}$ $\quad e \in \mathrm{E_B} \backslash \{|\mathrm{E_B}|\}, \mathrm{b} \in \mathrm{B}, s \in \mathrm{S}, t \in \mathrm{T} \backslash \{1\}$ (7)

$I_{1,\mathrm{t},b,e} = I_{1,t,r,e}$ $\quad e \in \mathrm{E_B} \backslash \{|\mathrm{E_B}|\}, \mathrm{b} \in \mathrm{B}, s \in \mathrm{S}, t \in \mathrm{T}_1$ (8)

$I_{s,t,\mathrm{r},e+1} = I_{s,t-1,\mathrm{r},\mathrm{e}}$ $\quad e \in \mathrm{E_R} \backslash \{|\mathrm{E_R}|\}, \mathrm{r} \in \mathrm{R}, s \in \mathrm{S}, t \in \mathrm{T} \backslash \{1\}$ (9)

$I_{s,\mathrm{t},\mathrm{r},\mathrm{e}} = I_{1,t,r,e}$ $\quad e \in \mathrm{E_R} \backslash \{|\mathrm{E_R}|\}, \mathrm{r} \in \mathrm{R}, s \in \mathrm{S}, t \in \mathrm{T}_1$ (10)

$I_{stfe+1} = I_{stfe} - A_{st-1fw}$ $\quad w \in \mathrm{W} \backslash \{|\mathrm{W}|\}, \mathrm{f} \in \mathrm{F}, s \in \mathrm{S}, t \in \mathrm{T} \backslash \{1\}$ (11)

$I_{s,\mathrm{t},f,e} = I_{1,\mathrm{t},f,e}$ $\quad e \in \mathrm{W} \backslash \{|\mathrm{W}|\}, f \in \mathrm{F}, s \in \mathrm{S}, t \in \mathrm{T}_1$ (12)

Transfers between farms: The number of piglets to be transferred to the rearing or fattening farms has to be entered the same week. Later on, after completing the expected growth for the phase, all of them exit at the same time. For this reason, piglets sent to rearing farms cannot exceed the total number of piglets weaned (i.e. of age $|\mathrm{E_B}|$) nor do the pigs starting the fattening phase exceed the number of pigs finishing the rearing phase:

$$A_{s,t,b} \le I_{s,t,|E_B|} \qquad s \in S, t \in T, b \in B \qquad (13)$$

$$A_{s,t,\mathrm{r}} \le I_{s,t,|E_R|} \qquad s \in S, t \in T, \mathrm{r} \in R \qquad (14)$$

$$A_{s,t,\mathrm{f}} \le I_{s,t,|E_F|} \qquad s \in S, t \in T, \mathrm{f} \in F \qquad (15)$$

Therefore, piglets to be transferred between farms goes onto the growing process in a new phase, in a new farm. Eq. 17 and 20 are non-anticipativity equations:

$$I_{\mathrm{s},t,r,|E_B|+1} = \sum_{b \in B} A_{\mathrm{s},t-1,b,r} \qquad s \in S, t \in T \backslash \{1\}, r \in R \qquad (16)$$

$$I_{\mathrm{s,t},r,\mathrm{e}} = I_{1,\mathrm{t},r,\mathrm{e}} \qquad s \in S, r \in R, \mathrm{t} \in \mathrm{T}_1 \qquad (17)$$

$$\sum_{r \in R} A_{\mathrm{s},t,b,r} = A_{\mathrm{s},t,b} \qquad s \in S, t \in T, b \in B \qquad (18)$$

$$I_{\mathrm{s},t,f,|E_R|+1} = \sum_{r \in R} A_{\mathrm{s},t-1,r,f} \qquad s \in S, t \in T \backslash \{1\}, f \in F \qquad (19)$$

$$I_{\mathrm{s,t},f,\mathrm{r}} = I_{1,\mathrm{t},f,\mathrm{r}} \qquad s \in S, f \in F, t \in T_1 \qquad (20)$$

$$\sum_{f \in F} A_{\mathrm{s},t,r,f} = A_{\mathrm{s},t,r} \qquad s \in S, t \in T, r \in R \qquad (21)$$

Transportation capacity: Constraints affecting transportation are related to the capacity of each truck. We need to distinguish the different type of transports namely between farms and to the abattoir. Animals sent to the abattoir are heavier than those transferred between farms. Therefore, different capacities or trucks may apply. Hence, the number of trucks used to transport culled sows to the abattoir will depend on the number of them π_{ba} in each sow farm and the trucks capacity ka_b as it states Eq 22. Pigs to be delivered to the abattoir from fattening sows, is constrained by Eq. 23 and the transport capacity of piglets between farms is bounded by Eq. 24:

$$\pi_{b,a} \leq Nka_{\mathrm{s},b} \cdot ka_b \qquad s \in S, b \in \mathrm{B} \qquad (22)$$

$$A_{\mathrm{s},t,f} \leq Nka_{\mathrm{s},t,f} \cdot ka_f \qquad s \in S, t \in T, f \in \mathrm{F} \qquad (23)$$

$$A_{\mathrm{s},t,h_1,h_2} \leq Nkg_{\mathrm{s},t,h_1,h_2} \cdot kg_{h_1} \qquad \mathrm{s} \in S, t \in T, h \in H - F \qquad (24)$$

Operational constraints: The number of piglets born alive will depend of the number of sows in the state i where farrowing occurs and the average litter size $LS_{b,i}$:

$$I_{\mathrm{s},t,b,1} \leq \sum_{i \in S_g \subset S} \pi_{b,i} \cdot LS_{b,i} \qquad s \in S, t \in T, b \in \mathrm{B} \qquad (25)$$

Where Sg∈S represents the set of reproductive states of a sow with a farrowing [11,13].

Constraints used to maintain a minimum stock in rearing and fattening farms are defined by scenarios (Eq. 26, 27). The minimum stock Jr_e and Jf_e is calculated previously keeping into consideration the production capabilities of the sow farms. Note these constraints are only valid for the group of scenarios in the last stage of the model:

$$\sum_{r \in R} I_{s,|T|,r,e} = Jr_e \qquad s \in S, r \in R, e \in E_R \qquad (26)$$

$$\sum_{f \in F} I_{s_s,|T|,f,e} = Jf_e \qquad s \in S, f \in F, e \in E_F \qquad (27)$$

Data and scenario generation

The data used to perform the empirical analysis correspond to an average size of typical companies in the pork sector in Spain. In that case we consider seven sow farms, twenty rearing farms and 124 fattening farms. Then Pigs produced, the piglets stay 4 weeks in lactation period on sow farms, 6 weeks in rearing farms and a

maximum of 18 weeks in fattening farms to complete the process. In this example, The marketing time window in fattening farms to transport pigs to the abattoir ranged from week 15 to 18 of the fattening period. The weekly cost per animals and week was 1.875€ at for sows farms, 2.66€ at for rearing farms pigs and 4.832€ at for fattening farms pigs and week per each type of farm respectively. The averaged litter size at farrowing stage was considered for simplicity. The expected number of piglets weaned is was also a constant different depending on sow parity. Sows are replaced culled and sent to the abattoir after 9 farrowings. Unitary transportation cost is fixed at 1 €/km over all the production process. A 52 week time horizon (one year) has been considered. The first week corresponds to the first stage letting the following 51 weeks to the second stage. This partition is motivated because the operational decision regarding transfers of animals and deliveries to the abattoir is taken week by week. Therefore, the second stage represent the uncertainty of prices affecting first stage decisions.
The model is formulated with 12 scenarios with different sales prices. Scenarios' generation is based on the weekly sales prices in Mercolleida's auction market, in Lleida (Spain) from the years 2010 to 2013. Figure 1 shows the real series of weekly sales prices. It is observed seasonality in prices, which tends to decrease at the end of each year and increase in the middle of the year. The Holt-Winters method [6,20] has been used to generate a forecast using Minitab Software. After that, 12 scenarios have been considered increasing and decreasing the weekly base prices a 2%, 4%, 6%, 8%, 10% and 12% respectively. Figure 2 shows the next two years forecasted pork prices and the set of scenarios.

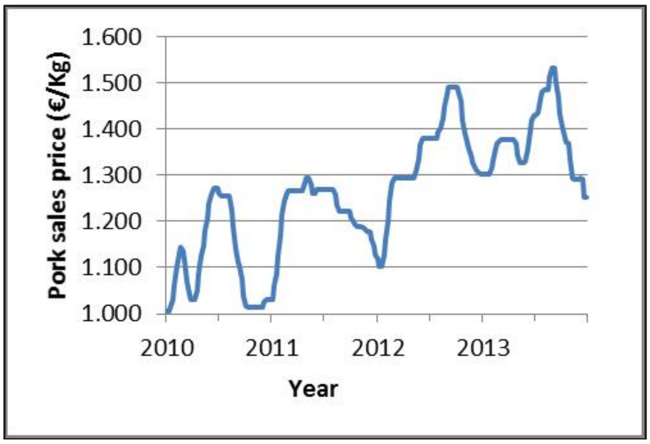

Figure 1. Weekly pork sales prices from jan-2010 to dec-2013

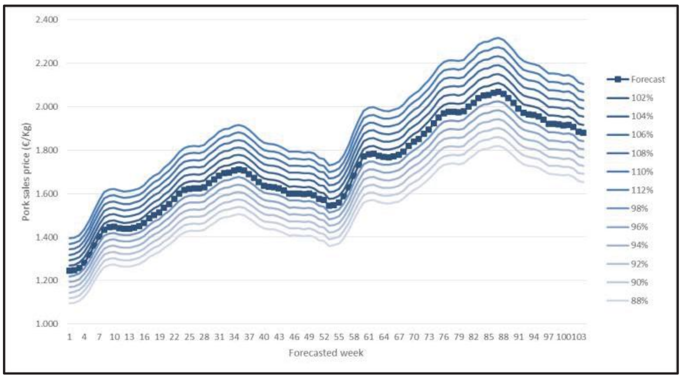

Figure 2. Two years forecast for the pork's price kilo and its scenarios (±2%, ±4%, ±6%, ±8%, ±10%, ±12%)

RESULTS AND DISCUSSION

To develop the model, the modeling language ILOG OPL has been used and CPLEX v12.4 solved the model in a Pentium 12 CPU's at 2.1GHz each one and 48 GB RAM. The execution of the model does not find a solution after 48 hours.

To improve the performance, some changes were analyzed. A first attempt was to reinforce the formulation of the model by restricting the deliveries of animals between certain farms depending of the distance, establishing a minimum batch of animals to be transferred or by fixing routes for the deliveries between farms. Afterwards, some computational experiments were performed to explore computational savings associated. The first experiment was to relax the integral property of the decision variables representing animals and trucks. Table 1 shows the results obtained. As it can be observed, the amount of time is considerably reduced when a full relaxation is considered.

Table 1. Computational and solution results comparative relaxing and decision variables relaxation (in thousands)

	All Integer	Trucks as continuous	Animals as continuous	All continuous
# Integer Variables	4,862	1,674	3,187	--
# Continuous Variables	--	3,187	1,674	4,862
Constraints	4,072	4,072	4,072	4,072
Solving time	>48 hours	>48 hours	>48 hours	1,502 s.

Taking in consideration the resolution time and working with continuous variables, some other experiments has been done simulating the model behavior time depending on how the operational decisions will be taken. Tactical decisions are mainly related to transport and farm capacities. As mentioned, decisions are generally taken on a weekly basis. Therefore, having one week in the first stage is enough to allow them to take these decisions. However, it is possible that operational decisions in some companies are not taken on a weekly basis or may require enlarging the stability of first stage decisions. Hence, in a second computational experiment we modify the number of weeks in the first stage of the model from one week until four weeks (1 month) but maintaining the 52 weeks in the total time horizon.

Finally, a last computational experiment was performed considering different time horizons. An increment in the time horizon increases the size of the model in terms of variables. The impact on first-stage decisions is also of interest and one week at the first-stage was considered in all instances. The time horizon was ranged from 52 weeks (1 year) to 80 (more than 1.5 years) to explore both the impact on solving time and first stage indexes. When the model was solved with a time horizon of 80 weeks instead of 52 the execution time increased by 400%. A big increment in the execution time was observed between weeks 64 and 68. Considering a time horizon of 85 weeks, an increment in the execution time similar to that of 68 weeks was observed. However, the increments in the resolution time beyond week 85 are maintained. Results reported for the first week corresponding to the operational decisions always remained the same. With this respect, the time horizon seems to not affect to the first-stage decisions.

CONCLUSIONS

A two-stage stochastic linear model has been formulated representing an average PSC company in Spain. The biggest concern was how to achieve a reasonable computational time for practical purposes due to the size of the model and the large number of integer variables. Instances of the model intended for practical purposes have been solved successfully allowing pig managers to improve operational decisions. Uncertainty in sales prices was used to generate different scenarios to be taken into account by first-stage decisions. As result of the empirical analysis, operational decisions (short-term decisions) are computed in a reasonable time. In this sense, some relaxation over integer variables seems mandatory to achieve reasonable solving times; in particular, inventories of animals can be relaxed. Relaxing the number of trucks is more questionable, but fixing as integer the trucks at the first-stage and relaxing these variables at the two-stage seems reasonable to get a more accurate transportation cost. Furthermore, a first-stage of one week is preferred. More weeks at the first stage do not affect operational decisions nor do optimal policies but impacts negatively in the execution time. In addition, and due the model takes into consideration all the phases of the pig production process, it can be also used to explore strategic decisions. However, we show that the execution of the model with more than one year and a half can be penalized due the execution time needed to solve it. Furthermore, the enlargement of the model considering larger time horizons compromise practical computational times. From our results, a planning time horizon of one year (52 weeks) produce enough good first-stage decisions that have to be recalculated in a rolling time horizon week by week. Other extensions of the model like the addition of new farms or more scenarios imply an increment in the size of the model and penalize the solving time. Therefore, the modeling of this kind of complex problems opens new research issues that must be tackled in future.

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